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**Robert Charles Krulish
and
Carl Stevens Mathews**

A MICROWAVE, DIRECTION-MODULATED,
HYPERBOLIC, NAVIGATIONAL SYSTEM

by

ROBERT CHARLES KRULISH
B.S., U.S. Coast Guard Academy
(1946)

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Robert Charles Krulish
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B.S., U.S. Coast Guard Academy
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CARL STEVENS MATHEWS
B.S., U.S. Coast Guard Academy
(1948)

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
DEGREE OF NAVAL ENGINEER

at the

MASSACHUSETTS INSTITUTE OF
TECHNOLOGY
JUNE, 1954

ABSTRACT

A MICROWAVE, DIRECTION MODULATED, HYPERBOLIC
NAVIGATIONAL SYSTEM

Robert C. Krulish

Carl S. Mathews

Submitted to the Department of Naval Architecture and Marine Engineering on 24 May 1954, in partial fulfillment of the requirements for the degree of Naval Engineer.

The object of this thesis is to delineate a navigational system capable of prescribing an arbitrary locus, such as a winding harbor channel, with an accuracy which will allow a navigator to negotiate such a channel safely in periods of darkness, fog, or low visibility; and to evaluate usefulness of system devised in solution of the navigational problem.

The time difference in receipt of signals simultaneously transmitted from two precisely located stations determines a hyperbolic line of position. If a delay is introduced between transmitted signals, the time difference for a certain line of position can be made arbitrary. If the radiation pattern of one station is made highly directional, the time difference may be made arbitrary on a small segment of a hyperbolic line of position. If the beam antenna is rotated, and a continuous variable delay is introduced which is a function of angle and an arbitrary line, the time difference in receipt of signals can be made to define the arbitrary line. This is a Microwave, Direction-Modulated, Hyperbolic Navigational System.

Procedures are developed for determination of system design parameters for application to any specific harbor, and for estimation of system error in this application.

These procedures are used, for determination of details for Boston Harbor, and are evaluated experimentally.

Results show that, in Boston Harbor, the system will allow a ship to stay within approximately 150 feet of the center of the safe navigable channel.

There were no peculiarities of Boston Harbor apparent which would indicate that accuracy of the same order could not be expected for numerous other applications.

Thesis Supervisor: Earl W. Keller, S.M.
Assistant Professor of Electrical Engineering

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Q. 23. 2013

Cambridge, Massachusetts
May 24, 1954

Professor Leicester F. Hamilton
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the
degree of Naval Engineer, we submit herewith a thesis
entitled, "A Microwave, Direction-Modulated, Hyper-
bolic Navigational System."

Respectfully,

ACKNOWLEDGMENT

The authors would like to express their appreciation to Prof. Earl W. Keller, S.M. of M.I.T., who, as thesis advisor, was helpful throughout the progress of the work, and to Paul W. Murray, Project Technician of the Communications Laboratory, whose able assistance was indispensable to successful completion of experiments.

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CHAPTER 1

INTRODUCTION AND SUMMARY

1.1 Aids to Navigation and Homing Devices

The problem of navigation has invited the interest of many and the life work of some in its development from a mystical art to the exact science we know today. One of the most important problems of navigation is position finding. If the navigator is in familiar territory, landmarks or other physical objects might give him a key to his position. Celestial navigation methods extend the scope of "familiar territory" practically to the entire surface of the earth. However, weather conditions or poor visibility sometimes make use of these natural aids impossible. Artificial devices of various kinds---lights, fog horns, bells, buoys---were used as navigational aids early in maritime history. These devices are, however, limited in effectiveness by the same factors which limit use of natural aids.

With the advent of radio came a new era in the fields of navigation. Beginning early in this century, radio became the tool for rapid development of new aids to navigation which were less dependent on weather and visibility for accurate, dependable service.

World War II gave tremendous impetus to the development of entire new systems of navigation which enabled ships and aircraft to determine their position accurately on trips of thousands of miles without seeing a single star or landmark. The configurations or patterns of lines of position given by these systems (reduced to simpler terms of plane rather than spherical geometry) have taken three basic forms:

able to use of natural aids.

[illegible]

- (1) Straight lines,
- (2) Circles, and
- (3) Hyperbolas.

Aircraft beacons, shoran, and loran respectively furnish these three types of pattern.^{1*} Position finding by use of these methods resolves into the determination of two or more lines of possible position.. The intersection of these lines of position determines the actual position of the craft. If a line of position obtainable from one of these systems happens to pass through or near the navigator's destination, the problem of navigation resolves into staying on the line of position through the destination, and obtaining occasional cross checks of position along that line. This process is called homing. Present systems can be used successfully for homing, provided the course to be followed is a straight line, a circle, or a hyperbola. Ships and aircraft can home on one or more of these systems to a harbor entrance or to the vicinity of an airstrip. Here a local system is required to give the navigator or pilot information which would allow him to negotiate a restricted harbor channel into port, or maintain a proper glide path to a landing. The system problem for aircraft has been solved satisfactorily by several means and will not be discussed here.² The surface problem is more complicated, and, to date, has received little attention.

1.2 Surface Homing Problem in Harbors and Solution by Proposed System

Define the surface system problem as follows:

- I. Coverage from outer harbor (or terminus of other homing system) to inner harbor.

* All superscripts refer to references listed in the Appendix.

- II. Rapid, accurate, and continuous indication of deviation from safe navigable waters.
- III. Satisfactory operation at night, in fog, and in periods of low visibility.

In addition to these requirements, it would be desirable to have the following system characteristics:

1. Simple, low-cost receiving and indicating equipment.
2. Minimum accuracy dependence on receiving and indicating equipment.
3. Minimum interference with present radio navigational systems, commercial and military communications systems, and present electronic installations.
4. Simple, reliable, stable transmitting and control elements.
5. Provisions for wartime security and adaptability to military problems.

Consider more closely the three major requirements. Requirement I limits the range of operation or coverage of area to a radius of five to twenty miles. A radio device of moderate power could give satisfactory performance at all frequencies up to 10,000 MC. Requirement III could also be met by a similar device. Requirement II, therefore, specifies the major portion of the solution which is not readily apparent.

The "safe navigable waters" of a harbor channel are not, unfortunately, a single straight line, circle, or hyperbola over a large portion of the channel length. The system, therefore, must establish lines of position which are arbitrary, and which follow the line of the channel in any harbor where the system is installed. The accuracy of transverse position in the channel should be such that the navigator can readily determine the

1. The first of these is the fact that the system is not a simple one, but a complex one, involving many different factors and many different people.

2. The second is the fact that the system is not a static one, but a dynamic one, which is constantly changing and evolving.

3. The third is the fact that the system is not a closed one, but an open one, which is constantly interacting with the outside world.

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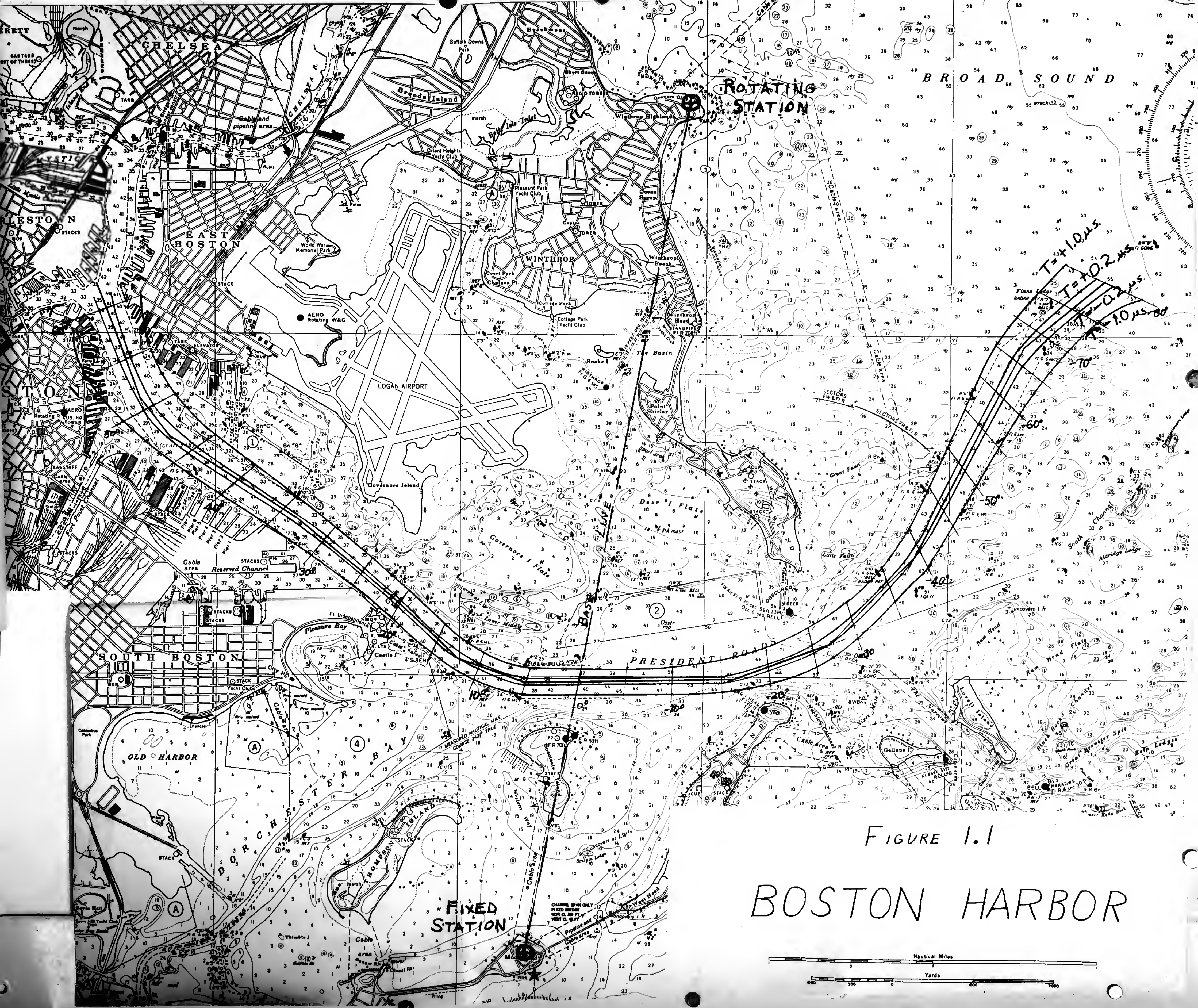


FIGURE 1.1
BOSTON HARBOR

safety of his position.

Several types of systems might be devised to give a satisfactory solution to the problem. A straight-line system using one beacon for each straight-line segment of a channel could possibly be used, but the number of beacons required would be large. Circular or hyperbolic systems such as Shoran or Loran could be modified to give position determination to required accuracy, but these systems would require extremely complex computing devices in the receiver to enable following an arbitrary line.

Present hyperbolic navigational systems, such as Gee, and Loran transmit accurately synchronized pulses from two stations separated geographically by a specified distance. A receiver at a point in the coverage area receives one signal before the other. The time difference in receipt of the two signals determines a hyperbolic line of position. This type of system seems to be most readily adaptable to requirements of the problem. These systems use omnidirectional antennas, and cover the entire service area with each transmission.

If the radiation pattern of one of the stations were to be changed to a narrow beam, the coverage area would be restricted to the sector of the beam, and the pattern of lines of position would be segments of the normal hyperbolic pattern. Furthermore, if a time delay is introduced in the transmission of pulses from the beam antenna, the modified time difference in receipt on a segment would be the sum of the normal time difference and the introduced delay. Thus, any segment may be made the zero time difference segment by introducing the proper delay. A course line passing through the narrow beam will intersect a hyperbolic segment. By introducing proper delay, the intersection could be made a point of zero time delay.

The beam antenna could be rotated, a small amount. The coverage area would then be another sector, on a different bearing from the first sector. Here, also, a delay could be introduced to make a point on course line a point of zero time difference. If, now, the beam antenna is rotated continuously, and the proper delay is introduced on each bearing, the course line becomes the zero time difference line.

In further discussion of the system, the zero time difference line will be made to follow a specified track. The transmitter with an omni-directional or very broad radiation pattern will be called the "Fixed" station. The other transmitter with a very narrow beam antenna and rotating at constant speed will be called the "Rotating" station.

The signals to control the two transmitters will originate in a "Timer" located at the Rotating station. The timing signal for the Rotating station will be delayed by a function of the antenna rotation, such that the signals from the Fixed and Rotating stations will arrive simultaneously at the intersection of the rotating antenna beam and track line. Thus at any point on the track line the time difference reading will be zero.

The system described differs from the normal hyperbolic system in two basic ways:

- (1) The radiation pattern from one of the stations must be highly directional, and
- (2) A variable delay which is a function of bearing and course position is introduced between pulses transmitted from the two stations.

Since high directivity is required, a microwave system would be most practicable. The resultant system is a "Microwave, Direction-Modulated,

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Hyperbolic Navigational System." Such a system is capable of generating arbitrary loci of constant time difference, and might be used to obtain the solution to the problem stated. For system components see Fig. 1.2.

1.3 Procedure

The derivation of governing equations was the first step in the development of the system. Before proceeding further it was necessary to derive equations defining system parameters, and to determine by analysis of these equations, what factors would control the accuracy of the system.

To transfer from the realm of theory to the realm of reality, it was necessary to obtain an idea of orders of magnitude of system parameters for a particular case. For this investigation, Boston Harbor was chosen. Tracklines were laid out on the chart from the sea-buoy to Charlestown Navy Yard. Sites were chosen for the transmitting stations on the basis of system criteria and topography (see Fig. 1.1). With the equations derived, and with information obtained from application to Boston Harbor, specifications for critical components were determined.

Experimental work was conducted to determine the practicability of equipment specifications. The work was carried out in two phases:

- (1) Test of critical components without RF link.
- (2) Test of critical components with RF link.

The first phase of the test was conducted to determine timer accuracy, and to explore the most practicable methods of indication. Significant additional information was gained from these tests concerning the effect on accuracy of spread in time difference due to beamwidth of the rotating antenna, and the effect of intermittent signals from the Rotating station

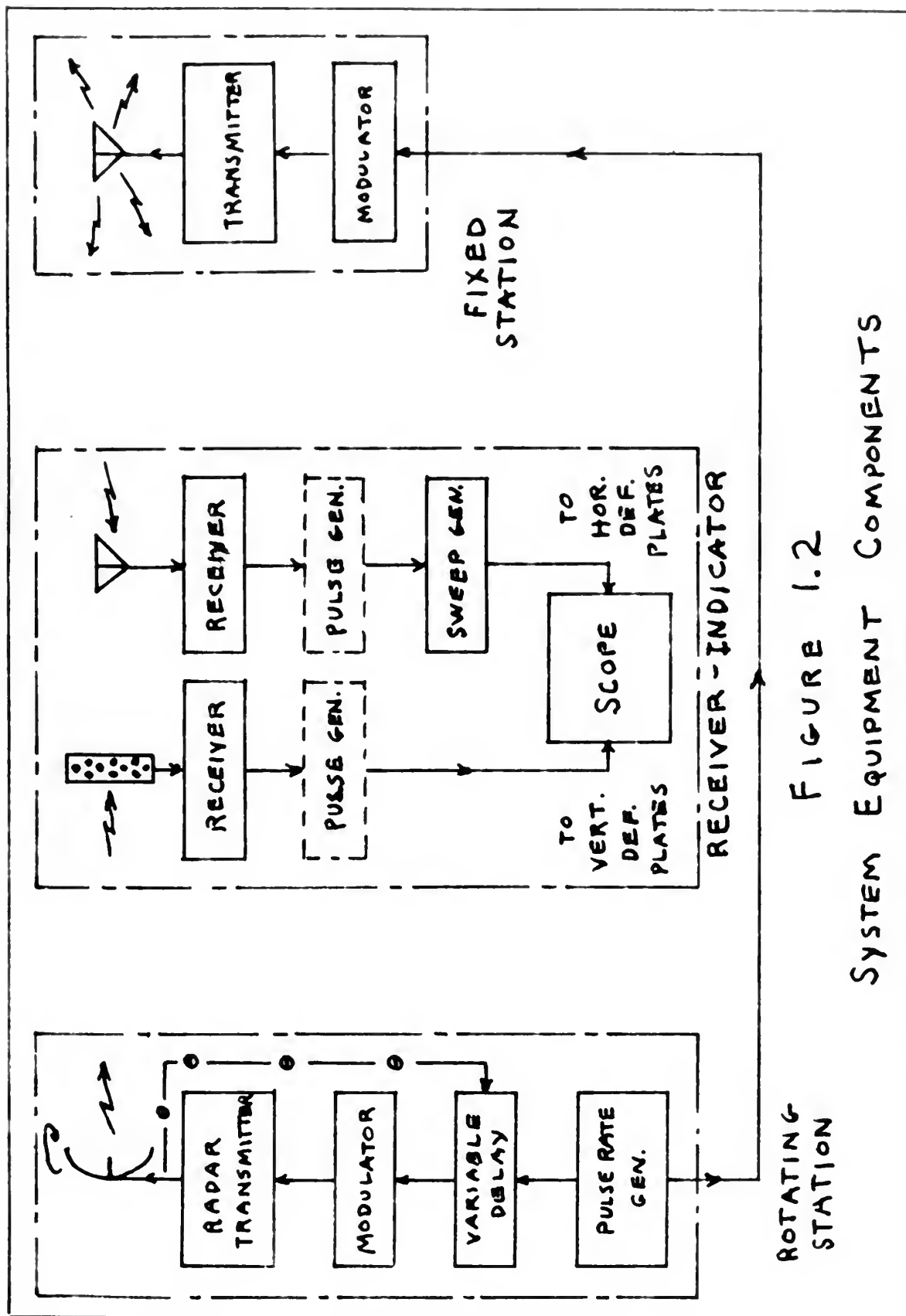


FIGURE 1.2
SYSTEM EQUIPMENT COMPONENTS

(occurring once every revolution of the antenna). The probable readable time difference accuracy as determined by these experiments shows the distance accuracy of the system by application of derived equations.

The second phase of the tests was conducted to confirm results of the first phase. Equations had been derived relating accuracy at any point in the coverage area to accuracy along the base line. Therefore, a test along the baseline was a test of the entire system.

1.4 Results and Conclusions

The probable system error expressed in time difference was found to be 0.1 us. This time difference is equivalent in distance to approximately ± 50 feet on the base line or to a maximum of ± 150 feet anywhere in the coverage area. This type of channel definition is more than satisfactory for Boston Harbor, and should be obtainable for many other harbors. Therefore, we may conclude that the systems of this type would contribute materially to the solution of a navigator's problem of negotiating harbor channels especially in periods of darkness and low visibility.

The proposed system error expressed in this difference was found to be 0.1 m. This difference is equivalent in distance to approximately 150 feet on the base line or to a maximum of 150 feet anywhere in the coverage area. This type of channel definition is more than satisfactory for local navigation, and would be obtainable for many other purposes. Therefore, we may conclude that the system of this type would be suitable for the solution of a navigator's problem of negotiating narrow channels especially in regions of darkness and low visibility.

1.4. Results and Conclusions

The proposed system error expressed in this difference was found to be 0.1 m. This difference is equivalent in distance to approximately 150 feet on the base line or to a maximum of 150 feet anywhere in the coverage area. This type of channel definition is more than satisfactory for local navigation, and would be obtainable for many other purposes. Therefore, we may conclude that the system of this type would be suitable for the solution of a navigator's problem of negotiating narrow channels especially in regions of darkness and low visibility.

DETERMINATION OF FUNDAMENTAL GEOMETRIC RELATIONS

The values required for the introduced delay can be obtained graphically from the normal hyperbolic pattern. In Figure 2.1, take a point on the trackline and read from the normal hyperbolic pattern the time delay which would be received at this point. When the antenna is directed at this point a cancelling delay must be introduced in the timer. A mathematical expression for the delay will, however, be more convenient and lend itself more readily to interpretation and evaluation.

2.1 Derivation of Equation for Introduced Delay

The equations are derived in terms of polar co-ordinates about the Rotating Station, and on the assumption that within the region of operation the earth's surface can be considered as a plane. Figure 2.2 illustrates and defines the necessary terms.

$$l = L \sqrt{1 - 2a \cos\theta + a^2} \quad (1)$$

Let there be a timer located at R, which at time t will send a trigger to the transmitter at F. The timer will trigger the transmitter at R at time $(t + d)$, where d is defined as the introduced delay.

Thus one signal will start from R at time t , and travel from R to F to p. This signal will arrive at p at time t_1 where

$$t_1 = t + \frac{L (1 + \sqrt{1 - 2a \cos\theta + a^2})}{c}, \quad (2)$$

and c is the velocity of light.

The other signal will start from R at time $(t + d)$ and travel directly

The values of τ for the proposed delay can be obtained from the following expression. In Figure 2.1, take a point on the surface and from the normal hyperbolic pattern the time delay τ would be measured as this point. When the antenna is directed at this point a cancelling delay will be introduced in the time delay. The expression for the delay will, however, be more convenient and lead itself more readily to interpretation and evaluation.

2.1 Derivation of equation for introduced delay

The equations are derived in terms of polar co-ordinates about the Rotating Station, and on the assumption that within the region of operation the earth surface can be considered as a plane. Figure 2.2 illustrates and defines the necessary terms.

(1)

$$r = \sqrt{R^2 + 2Rd \cos \theta + d^2}$$

Let there be a time interval t , which is time t will send a trigger to the transmitter at R . The time will travel the distance R at time $t + t_1$, where t_1 is defined as the propagation delay. Then one signal will come from R at time t , and travel from R to P at time $t + t_1$. This signal will arrive at P at time $t + t_1 + t_2$ where

(2)

$$t_2 = \frac{1}{c} \sqrt{R^2 + 2Rd \cos \theta + d^2}$$

and c is the velocity of light. The other signal will come from R at time $t + t_1 + t_2$ and travel distance

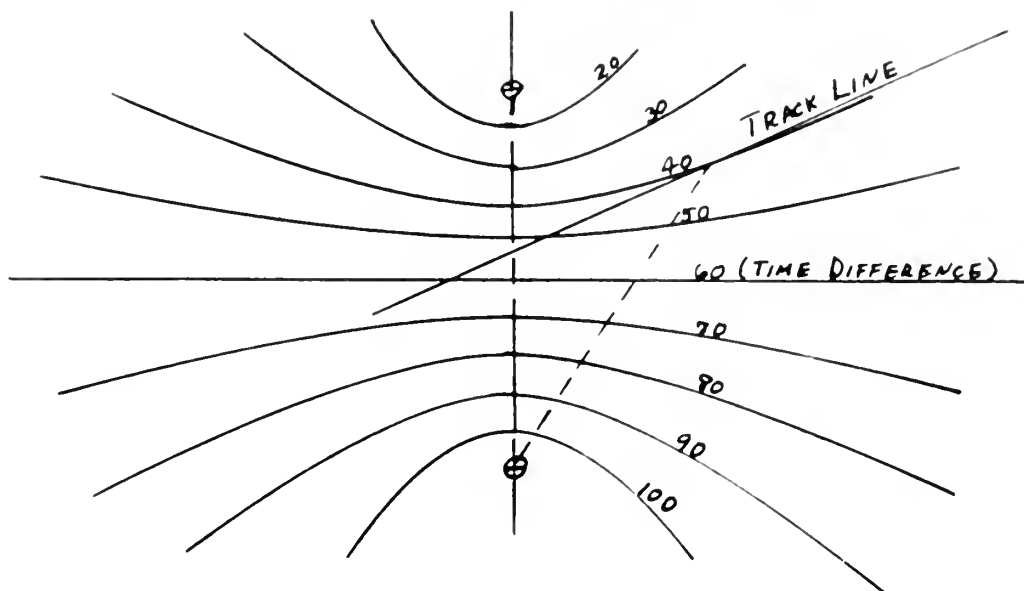
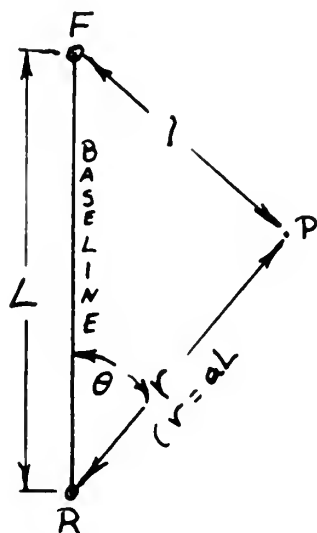


FIGURE 2.1
NORMAL HYPERBOLIC PATTERN



F - FIXED STATION
R - ROTATING STATION
L - BASELINE LENGTH
P - POINT (r, θ)

$$a \equiv r/L$$

FIGURE 2.2
COORDINATE SYSTEM



to p; arriving at time

$$t_2 = t + d + \frac{aL}{c} . \quad (3)$$

The time difference reading (T) will be $t_1 - t_2$.

$$T = \frac{L(1 - a + \sqrt{1 - 2a \cos\theta + a^2})}{c} - d, \quad (4)$$

for any point (a, θ) .

If it is desired to have $T = 0$ for a point (a, θ) on the trackline then,

$$d = \frac{L(1 - a + \sqrt{1 - 2a \cos\theta + a^2})}{c}. \quad (5)$$

2.2 Geometrical Accuracy

To determine the change in T for a small movement off the trackline along a radius from R, take the partial derivative of Equation 4 with respect to a. Note that d is a constant determined by Equation 5.

$$\frac{\partial T}{\partial a} = \frac{L}{c} \frac{a - \cos\theta}{\sqrt{1 - 2a \cos\theta + a^2}} - 1. \quad (6)$$

Rearranging and noting that $\partial T / \partial a = \partial T / \partial r$, since $r = aL$,

$$c \frac{\partial T}{\partial r} \equiv \text{R.S.} = \frac{a - \cos\theta}{\sqrt{1 - 2a \cos\theta + a^2}} - 1. \quad (7)$$

This defines the radial sensitivity (R.S.), a nondimensional term.

A dimensional term, Radial Divergence (R.D.) may be useful.

$$\text{R.D.} \equiv \frac{c}{\text{R.S.}} \quad (\text{ft. per us.}) \quad (8)$$

Figure 2.3 shows the locus for several values of Radial Sensitivity.

(1)

$$\frac{d\theta}{dt} = \frac{v}{r}$$

where v is the velocity of the particle and r is the radius of the circular path.

(2)

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\frac{d\theta}{dt} r} = \frac{2\pi}{\frac{d\theta}{dt}}$$

for any point (θ, r) .

If we assume that the particle is moving in a circular path of radius r and the angular velocity is ω , then

then

(3)

$$d = \frac{L}{\omega} = \frac{L}{\frac{d\theta}{dt}} = \frac{L}{\frac{v}{r}} = \frac{Lr}{v}$$

2.2 Geometrical Approach

To derive the expression for T for a small movement off the trackline along a radius from θ , take the partial derivative of Equation 1 with respect to θ . Note that θ is a constant determined by Equation 2.

(4)

$$\frac{\partial T}{\partial \theta} = \frac{1}{\omega} \frac{\partial \omega}{\partial \theta} = \frac{1}{\omega} \frac{v}{r} \frac{\partial r}{\partial \theta} = \frac{1}{\omega} \frac{v}{r} \frac{dr}{d\theta}$$

where $\frac{dr}{d\theta}$ is the derivative of the radius with respect to the angle θ , and $\frac{\partial \omega}{\partial \theta} = \frac{v}{r} \frac{\partial r}{\partial \theta}$ since $v = \omega r$.

(5)

$$\frac{\partial T}{\partial \theta} = \frac{1}{\omega} \frac{v}{r} \frac{dr}{d\theta} = \frac{1}{\omega} \frac{v}{r} \frac{dr}{d\theta}$$

where $\frac{dr}{d\theta}$ is the derivative of the radius with respect to the angle θ , and $\frac{\partial \omega}{\partial \theta} = \frac{v}{r} \frac{\partial r}{\partial \theta}$ since $v = \omega r$.

where $\frac{dr}{d\theta}$ is the derivative of the radius with respect to the angle θ , and $\frac{\partial \omega}{\partial \theta} = \frac{v}{r} \frac{\partial r}{\partial \theta}$ since $v = \omega r$.

(6)

$$\frac{\partial T}{\partial \theta} = \frac{1}{\omega} \frac{v}{r} \frac{dr}{d\theta} = \frac{1}{\omega} \frac{v}{r} \frac{dr}{d\theta}$$

where $\frac{dr}{d\theta}$ is the derivative of the radius with respect to the angle θ , and $\frac{\partial \omega}{\partial \theta} = \frac{v}{r} \frac{\partial r}{\partial \theta}$ since $v = \omega r$.

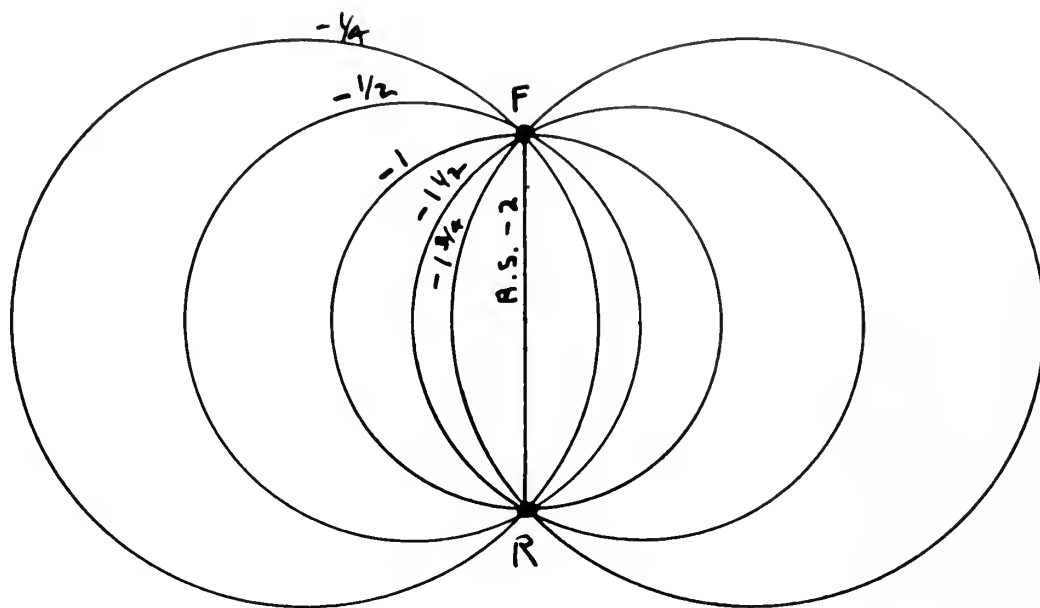
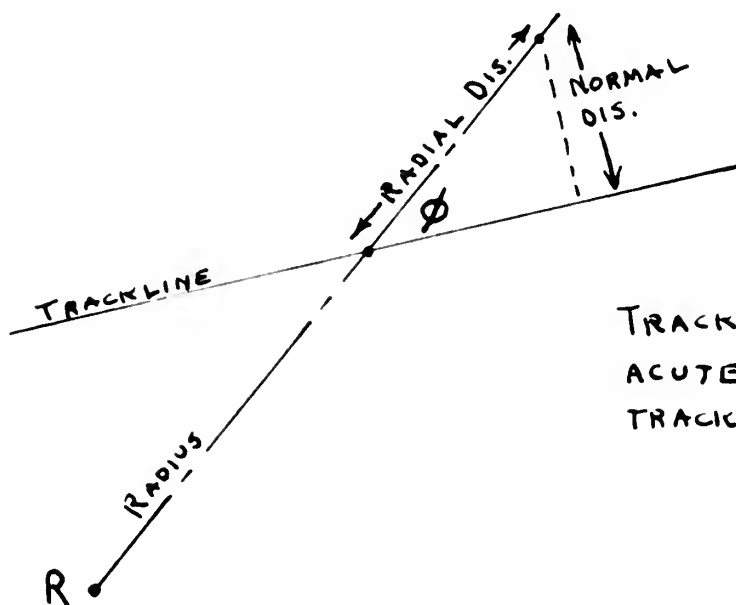


FIGURE 2.3
LOCI OF RADIAL SENSITIVITY



TRACK ANGLE (ϕ) IS
ACUTE ANGLE BETWEEN
TRACKLINE AND RADIUS.

FIGURE 2.4
EFFECT OF TRACK ANGLE ON P.D.

The details of determination are in the appendix. It can be shown that on the baseline the Radial Divergence will be 492 ft./us.; and that within a circle drawn with the baseline as a diameter (hereafter called the Baseline Circle), the maximum value will be 384 ft./us.

The Radial Divergence gives the best indication of the expected geometrical accuracy for an arbitrary trackline. If the trackline is specified, a more significant figure is the distance normal to the trackline per microsecond of delay. This is expressed as the Pattern Divergence (P.D.). For an angle ϕ between radius and trackline,

$$P.D. = R.D. \sin\phi \text{ (ft./us.)} \quad (9)$$

Figure 2.4 shows how this was obtained from Equation 8.

The Radial Divergence remains important in that it expresses the maximum value of Pattern Divergence independent of track angle.

It can be concluded that within the baseline circle the error, in distance normal to the trackline, will be less than 1,000 ft./us. error in time difference reading.

2.3 Factors Effecting Readable Accuracy.

It has been pointed out that the introduced delay will vary continuously with antenna rotation, and that the time delay received will vary over a range equal to the variation in d per beamwidth. The effect of this variation will depend on whether or not it is linear, and whether the indicator reads the average, the mean, or the extreme. The maximum effect would be to introduce an error equal to the variation in d for half a beamwidth. Let the error due to variation in d be e_d .

An equation is developed in the Appendix for e_d on the basis of linear variation of d with θ , and assuming the error to equal half the

variation of d per beamwidth.

$$e_d = \frac{d(d)}{d\theta} \frac{B.W.}{2} \quad (10)$$

$$e_d = \frac{aL}{c} \left[\frac{\sin\phi}{\sqrt{1 - 2a \cos\theta + a^2}} + \left(\frac{a - \cos\theta}{\sqrt{1 - 2a \cos\theta + a^2}} \right) \cot\phi \right] \frac{B.W.}{2} \quad (11)$$

The range of variation in the delay for the complete pattern will effect the percentage accuracy required in the timer for a specified maximum error. Let this error be e_t .

The indicator will introduce an error which should be independent of system geometric parameters. Let this error be e_i .

The error in reading time difference will be the sum of these errors, and the error in distance off the trackline will be E , where

$$E = (e_d + e_t + e_i) \text{ P.D. (ft.)} \quad (12)$$

In general, the range of variation of delay should be indicative of variations per beamwidth. To the extent that this is so, both e_d and e_t will be dependent on the range of variation of delay.

2.4 Location of Stations

The system equations should furnish some information to serve as a guide in choosing sites for the transmitting stations.

As the introduced delay is to be a function of θ , it is apparent that the trackline must not intersect any radius from the Rotating Station more than once.

Figure 2.3 shows how the transmitting stations should be located, with reference to the service area, for high values of Radial Sensitivity.

$$\frac{1}{2} \left[\left(\frac{1}{2} \right)^n + \left(\frac{1}{2} \right)^{n-1} + \dots + \left(\frac{1}{2} \right)^1 + \left(\frac{1}{2} \right)^0 \right]$$

The sum of the series is $\frac{1}{2} \left[\frac{1 - \left(\frac{1}{2} \right)^{n+1}}{1 - \frac{1}{2}} \right] = 1 - \left(\frac{1}{2} \right)^{n+1}$.

Let $x = \frac{1}{2}$. Then the sum of the series is $1 - \left(\frac{1}{2} \right)^{n+1}$.

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(51)
$$\frac{1}{2} \left[\left(\frac{1}{2} \right)^n + \left(\frac{1}{2} \right)^{n-1} + \dots + \left(\frac{1}{2} \right)^1 + \left(\frac{1}{2} \right)^0 \right]$$

In general, the sum of the series is $1 - \left(\frac{1}{2} \right)^{n+1}$.

Let $x = \frac{1}{2}$. Then the sum of the series is $1 - \left(\frac{1}{2} \right)^{n+1}$.

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It has been shown that the values of c can be obtained from the normal hyperbolic pattern. It can be seen from figure 2.1 that an attempt to keep the Pattern Divergence small by orienting the stations to give small track angles (as per Equation 9) would result in large variations in the introduced delay. The effect of this was discussed in connection with Equation 12. Although no positive connection between θ_d and range of d has been made; it can be pointed out that P.D. is a function of $\sin\phi$ and has a finite limit, while $d(d)/d\theta$ is a function of $\cot\phi$ and has no limit. It seems desirable to keep the variation in delay at a minimum.

Topography will determine the extent to which theoretical requirements can be followed.

The following general rules should lead to a satisfactory positioning of stations:

1. No radius from the Rotating station can intersect the trackline more than once.
2. Include as much of the trackline as possible within the baseline circle.
3. The mean track should correspond as closely as possible to the perpendicular bisector of the baseline.

There is a possibility that the connection between the two points is not direct, but that it is indirect, passing through one or more intermediate points. It is also possible that the connection is not at all, but that the two points are independent of each other. The following table shows the results of the investigation.

Results are as follows:

The following general rules should be followed in connection with the investigation:

1. The following rules should be followed:

1. No further investigation should be made unless the results of the investigation are satisfactory.
2. The investigation should be continued until the results are satisfactory.
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CHAPTER 3

APPLICATION OF THE PROPOSED NAVIGATIONAL SYSTEM TO BOSTON HARBOR

To give practical significance to the theoretical relations developed in the previous chapter, it is necessary to have specific values for the important variables. Boston Harbor is by no means typical of all harbors or restricted waters; the values determined for Boston Harbor will, however, be of a practical order of magnitude.

3.1 Determination of Parameters

3.1.1 Channel and Tracklines. Safe navigable tracklines were laid out on a Boston Harbor chart from the start of the dredged channel to Charlestown Navy Yard. These are shown on Figure 1.1.

The minimum width of the channel is 400 yards. For safe one way traffic it would seem desirable that the system give an indication of deviation from the track when the distance off is a maximum of 100 yards. For two way traffic the maximum allowable error would be 50 yards.

3.1.2 Location of Stations. Sites for the transmitting stations were selected in accordance with the relations determined in the previous chapter, insofar as topography would permit.

The Fixed Station as well as the Rotating Station was located to allow for line-of-sight transmission. Later development shows this is not an absolute requirement for the Fixed Station.

3.1.3 Data for Boston Harbor. Table I gives the essential data for the Boston Harbor system, determined for five degree increments of θ . More detailed data is given in Table III in the Appendix.

of restricted waters; the values obtained for bottom trawls will, however, be of a practical order of magnitude.

estimated to contribute 1.0

3.1.1 Channel and Trenches. This navigable freshwater area is on a lower level than the rest of the dredged channel to Chinese- town Bay. There are about 100 ft. and 1.1.

[illegible]

The above exhibits are being submitted herewith as evidence.

Sincerely,
J. Edgar Hoover

1. The first of these is the fact that the
the Bureau of the National Intelligence Agency
is not a single entity but a collection of
many different organizations, each with its
own set of interests and objectives. This
makes it difficult to coordinate the efforts
of the various agencies and to ensure that
they are all working towards the same goals.

[illegible]

and 10 to a normal weight with a 10% increase in body weight.

TABLE I

θ	a	d	$d/\cos.$	$R.S.$	$P.D.$	θ	$P.D.$
-30°	.535	47.2	.20	-.658	1457	60°	1293
-75	.520	46.3	.19	-.736	1336	66	1220
-70	.496	45.4	.18	-.838	1172	70	1100
-65	.480	44.5	.28	-.937	1050	82	1038
-60	.500	42.6	.35	- 1	984	77	959
-55	.510	41.0	.37	-1.078	914	70.5	861
-50	.530	38.9	.44	-1.146	851	67	790
-45	.553	36.6	.50	-1.213	811	61	708
-40	.582	33.9	.50	-1.275	771	56	639
-35	.602	31.6	.50	-1.354	726	69	677
-30	.627	28.9	.51	-1.433	686	67	632
-25	.648	26.5	.43	-1.519	648	84	645
-20	.656	24.6	.37	-1.638	601	81	594
-15	.665	22.8	.78	-1.762	558	88	556
-10	.665	21.8	.19	-1.882	522	87	521
- 5	.670	20.9	.19	-1.965	500	84	498
0	.680	19.90	.18	-2.00	492	78	482
5	.696	19.2	.2	-1.975	498	87	497
10	.695	20.1	.25	-1.857	530	89	529
15	.686	21.6	.37	-1.738	566	77	552
20	.673	23.8	.41	-1.615	609	80	600
25	.665	25.7	.35	-1.494	658	86	657
30	.666	27.3	.31	-1.371	716	89	716
35	.673	28.8	.18	-1.246	789	76	764
40	.691	29.1	.22	-1.120	879	69	820
45	.716	31.0	.32	-.987	997	64	895
50	.736	32.3	.33	-.880	1118	80	1100
55	.736	34.3	.41	-.806	1220	89	1220
60	.735	36.4	.42	-.738	1334	90	1334
	in.	us.	us./deg.		Ft/us.		ft/us.

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Note that the maximum value of the Pattern Divergence is 1.334 ft./microsecond. This means that the error in reading time delay must be limited to approximately 0.2 microseconds for one way traffic.

3.2 Specifications for Design of Equipment

3.2.1 Timer. The timer must be variable over the range from 18-48 microseconds. Simplicity in the timer is of secondary importance, so accuracy should be specified to practically eliminate it as a source of error. One tenth of the desired overall accuracy, or 0.02 us., should be adequate. Long time instability requiring monitored operation will be acceptable.

3.2.2 Transmitters and Antennas. The beamwidth from the Rotating station must be kept as small as possible. This requirement dictates that the frequency should be high in order to obtain a narrow beamwidth with an antenna of practical size. Use a frequency in the 3 cm. band as frequencies above this are effected too much by atmospheric conditions to be considered useable. Beamwidths of 0.1 degree can be obtained for microwave frequencies;³ but considering that the antenna must be rotated, and perhaps at a fairly rapid rate, a specification of 0.5 degree beamwidth seems more practical.

The antenna should be rotated as fast as is practical, considering the physical problem, and the transient response of the timer and control, which must vary the delay with antenna rotation.

The frequency for the Fixed transmitter should be such that a steep leading edge can be maintained without requiring unreasonable bandwidth in the receiver. It should also be at a frequency which will permit good rejection of signals in the 3 cm. band. The antenna should be omni-directional or broad enough to cover the entire service area.

1. The frequency of the signal should be such that a steady leading edge can be maintained without requiring unreasonable bandwidth in the receiver. It should also be at a frequency which will permit the rejection of signals in the 100-1000 Mc. band. The antenna should be omnidirectional or broad enough to cover the entire horizon.

1.2.2. Transmitter and Antenna

The transmitter and antenna must be capable of operating over the range from 10-100 Mc. The transmitter should be capable of operating at a frequency of 10-100 Mc. The antenna should be capable of operating at a frequency of 10-100 Mc. The transmitter should be capable of operating at a frequency of 10-100 Mc. The antenna should be capable of operating at a frequency of 10-100 Mc.

1.2.3. Receiver and Antenna

The receiver and antenna must be capable of operating over the range from 10-100 Mc. The receiver should be capable of operating at a frequency of 10-100 Mc. The antenna should be capable of operating at a frequency of 10-100 Mc. The receiver should be capable of operating at a frequency of 10-100 Mc. The antenna should be capable of operating at a frequency of 10-100 Mc. The receiver should be capable of operating at a frequency of 10-100 Mc. The antenna should be capable of operating at a frequency of 10-100 Mc.

1.2.4. Receiver and Antenna

The receiver and antenna must be capable of operating over the range from 10-100 Mc. The receiver should be capable of operating at a frequency of 10-100 Mc. The antenna should be capable of operating at a frequency of 10-100 Mc. The receiver should be capable of operating at a frequency of 10-100 Mc. The antenna should be capable of operating at a frequency of 10-100 Mc.

The power required from both transmitters will be dependent on the receivers used; it should be kept in mind that it is desirable to have the receivers as simple as possible.

Pulse widths and pulse repetition rate will be dependent on the type of indicator used.

3.2.3 Receiver. The receiver must have one channel, either broad band or automatically tuned, to receive signals in the 3 cm.band. The other channel should be tuneable to the frequency of the Fixed Station.

Receiving antennas for both channels should be broadband and omnidirectional.

Both channels should have sensitivity, and automatic gain control which will produce output signals of fairly constant amplitude for signal strengths experienced anywhere in the service area. Bandwidths must be sufficient to produce sharp leading edges on the output signals.

3.2.4 Indicator. The total allowable error has been specified as 0.2 us. The timer error has been specified as 0.02 us. The maximum variation in delay per beamwidth for Boston Harbor is 0.25 which can introduce a maximum error of 0.13 us. Therefore the indicator error must be less than 0.05 us. This accuracy must be maintained when signals from the Rotating Station are received once per revolution, and the spread in delay of signals received is 0.25 us.

[illegible]

CHAPTER 4

EXPERIMENTAL EVALUATION OF SPECIFICATIONS

The experimental procedure as outlined in Chapter 1 was divided into two phases. The first phase was Test of Critical Components with the RF. link.

4.1 Test of Critical Components Without RF Link

4.1.1 Timer Construction. The specifications as outlined in Chapter 3 required accuracy for the variable time delay in the order of 0.01 us. Although timers with this accuracy could be easily constructed with normal commercial practice, such a timer was not available for use in these experiments. Therefore, a simple phantastron circuit was built with design adapted from the ranging circuit of the SU radar. The range of time delay in this circuit was from 30 us. to 56 us. or 36 us. The delay potentiometer was a ten-turn helipot. The dial on the helipot was graduated in degrees. With ten-turns of 360 degrees each, the 36 us. variation in delay was divided into 3,600 parts. The resultant calibration was for 0.01 us. per degree of shaft rotation. With this simple circuit, the specified timer accuracy was easily obtainable. To balance the 20 us. minimum in the variable delay, a fixed delay was constructed to facilitate further tests.

4.1.2 Indicator Patterns. This study was made to determine best indicator pattern. The set-up used is shown by Figure 4.1. Both pulses are applied to vertical deflection plates, and the sweep is triggered from the first pulse received. Various pulse widths for Fixed and Rotating stations were studied with various ratios of pulse amplitude. The results

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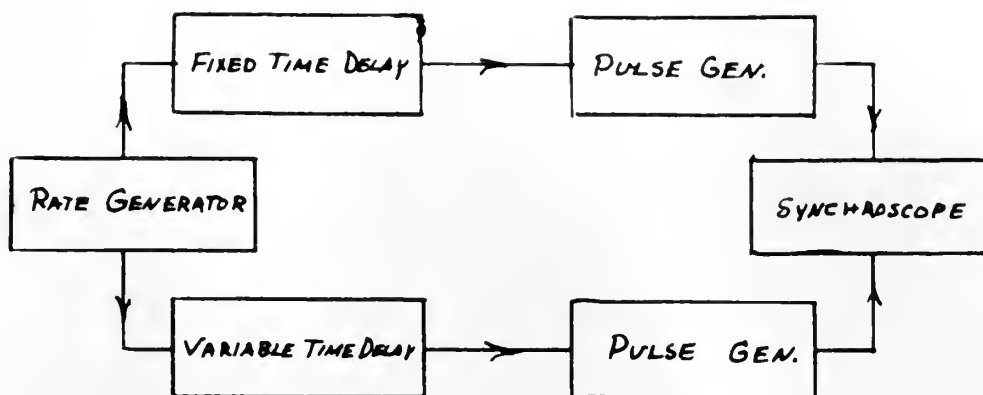


FIGURE 4.1

SIMPLIFIED DIAGRAM FOR CRITICAL COMPONENT
TEST WITHOUT THE R.F. LINK

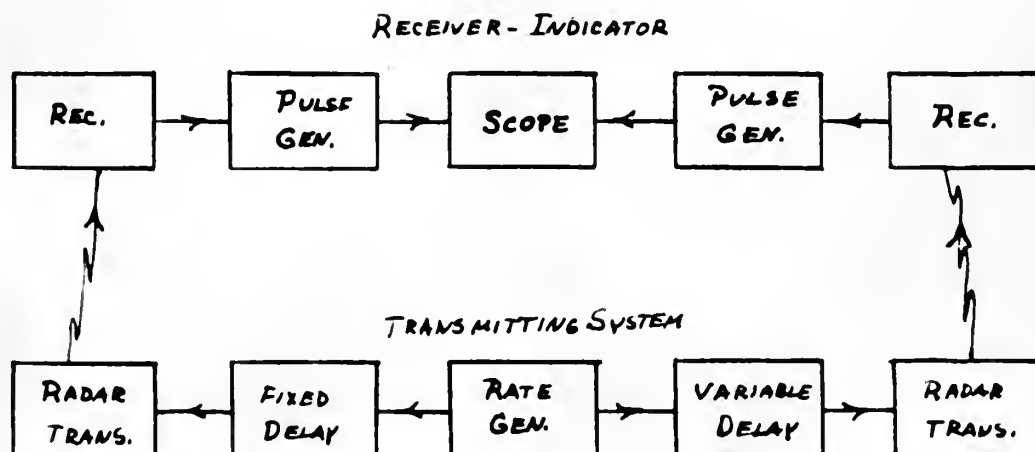


FIGURE 4.2

SIMPLIFIED DIAGRAM FOR CRITICAL COMPONENT
TEST WITH THE R.F. LINK



of these tests indicated that the Fixed station pulse length should be slightly greater than the total variation in time delay expected within the coverage area. For Boston Harbor, this length should be about 4.0 us. The Rotating station pulse length should be about 2.0 us. In our previous discussions, we have considered that $T = 0$. In order to facilitate matching, it is convenient to make $T = \text{constant} = \frac{1}{2}$ maximum variation of d , for all points in the coverage area. This is consistent with previous theory. With this type of presentation, the condition for matching was the condition where the center of the pulses of the two stations coincided. This type of matching is difficult to achieve because of difficulty in equalizing lengths of Fixed station pulse extending beyond the ends of the Rotating station pulse. Further, this type of matching is undesirable because pulse distortion at the transmitter and in the receiver is such that the sharp trailing edge of the signal is lost. This distortion makes matching of pulse centers difficult, if not impossible, without the use of auxiliary pulse generators in the receiver for each station.

Another type of matching which would utilize only the leading edges of pulses could be described as follows:

- (1) The signal from the Fixed station is received and triggers the sweep on the indicator.
- (2) The Rotating station signal is applied to the vertical deflection plates of the indicator scope, and, in the matched condition, appears with its leading edge at the center of the sweep.
- (3) The sweep length for operation in Boston Harbor should be about 4 us.

Tests with this type of indicator showed that average error in matching

was about one degree of shaft rotation on the timer. This corresponds to a timer-indicator accuracy of 0.01 us. This accuracy is slightly better than that obtainable with matching of pulse centers, and eliminates the need for one of the auxiliary pulse generators in the receiver. In order to obtain the proper video presentation it is necessary to have in the receiver either an extremely linear sweep with graduations on the scope face, or a crystal oscillator to provide marks on the sweep.

4.1.3 Effect of Spread. The causes of spread in time difference have been discussed in detail in earlier sections. In order to simulate this spread in static tests, a sinusoidal voltage was impressed on the timer so that the resultant time delay varied about the time delay set by potentiometer shaft. Matching of time difference was again accomplished as detailed above. Results of this test, (tabulated below) showed that (σ_1 σ_2) increased with increase in spread.

Continuous Signals without R.F. Link:

<u>Spread</u>	<u>Error</u>	
-0-	0.7°	0.007 us.
0.5 us.	0.8°	0.008 us.
1.0 us.	1.3°	0.013 us.

The precise functional relationship between spread and error cannot be formulated with this limited data. However, the order of magnitude is significant.

4.1.4 Effect of Sampled Data. In order to simulate intermittent receipt of signals from the rotating station, a key was placed in the trigger line of the Rotating station pulse generator. The key was closed at a rate corresponding to the rotation rate which might be expected in the

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Time	Speed
0.000 sec	0.000
0.001 sec	0.001
0.002 sec	0.002

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nal system. Extreme rates studied were 12 and 240 signals per minute. Error with high repetition rates was about the same as errors with continuous signals, and increased very slightly with rates as low as 60 per minute. As the rate was decreased beyond this point, error increased sharply. Results of test with varying Pulse Recurrence Rate from Rotating Station; sp. d, constant, at 0.5 us:

<u>P.R.R.</u>	<u>Error</u>	
Continuous	0.8°	0.008 us.
160-240/min.	1.1°	0.011 us.
60/min.	1.1°	0.011 us.
12/min.	2.4°	0.024 us.

It can be concluded that repetition rate should be kept as high as possible, but that rates as low as 60 per minute give acceptable results.

4.2 Test of Critical Components with RF Link

The second phase of the experimental work was conducted with an array of equipment indicated by Figure 4.2.

The equipment was set up for determination of the accuracy obtainable along the baseline. In previous work--derivation of equations-- we related the accuracy anywhere in the pattern to the accuracy on the baseline. Therefore, a test along the baseline, would test the accuracy obtainable anywhere in the coverage area of the system. In the first sequence of tests a constant time delay was set on the timer. The receiver unit was moved back and forth along the baseline, until the time difference was zero. The receiver was then moved off of the zero time difference point and returned to the indicated zero time difference point. The distance between successive indicated zero time difference

points was a measure of the accuracy with which a specified time delay would determine a position along the base line. The second part of the RF test was to vary the introduced time delay, leaving the receiver in the same position. The time delay was manipulated until a zero time difference was indicated. This test was intended to give reciprocal results to the first tests. The correlation of data obtained in these two tests would be a measure of the accuracy of reading obtainable along the baseline with the RF link.

The first part of this test indicates that maximum distance errors obtained from receiver movement were of the same order of magnitude as errors introduced by other system components. Results of this test are not significant in themselves.

The second part of the test showed that the value of $(e_i + e_t)$ with continuous signals was approximately 0.03 us. probable with the R.F. link. No spread was introduced, but instability of system components caused a spread of approximately 0.5 us. This instability would not be expected in components designed for this use, but was fortuitous in this test.

4.3 Evaluation of Experimental Results

The essence of the experimental tests is evaluation of the quantity $(e_i + e_t)$, with spread, intermittent signals, and the all essential system components.

The probable error $(e_i + e_t)$ through all system components but with continuous signals was 0.03 us. The effect of sampled data was shown to be negligible in comparison to this value for rotation rates of interest. The quantity e_t was found to be negligible. The causes of e_i were distributed throughout system components, and might be reduced in components designed for use in the system.

The first part of the paper is devoted to a description of the experimental setup and the results of the measurements. The second part is devoted to a theoretical analysis of the results. The third part is devoted to a discussion of the results and their significance. The fourth part is devoted to a conclusion.

The experimental setup consists of a system of two coupled pendulums. The first pendulum is a simple pendulum of length l_1 and mass m_1 . The second pendulum is a double pendulum of length l_2 and mass m_2 . The two pendulums are coupled by a spring with spring constant k . The equations of motion for the system are given by

$$m_1 \ddot{\theta}_1 + \frac{m_2}{2} \ddot{\theta}_2 + m_1 g \theta_1 + \frac{k}{2} (\theta_1 - \theta_2) = 0$$

$$m_2 \ddot{\theta}_2 + m_2 g \theta_2 + k (\theta_2 - \theta_1) = 0$$

where θ_1 and θ_2 are the angular displacements of the two pendulums from the vertical. The results of the measurements show that the system exhibits a rich variety of behavior, including periodic motion, quasi-periodic motion, and chaotic motion. The theoretical analysis shows that the system is Hamiltonian and that the equations of motion can be solved exactly. The discussion of the results shows that the experimental results are in good agreement with the theoretical predictions. The conclusion is that the system is a good example of a Hamiltonian system and that the equations of motion can be solved exactly.

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3.1. Theoretical analysis

The theoretical analysis of the system is based on the assumption that the two pendulums are coupled by a spring with spring constant k . The equations of motion for the system are given by

$$m_1 \ddot{\theta}_1 + \frac{m_2}{2} \ddot{\theta}_2 + m_1 g \theta_1 + \frac{k}{2} (\theta_1 - \theta_2) = 0$$

$$m_2 \ddot{\theta}_2 + m_2 g \theta_2 + k (\theta_2 - \theta_1) = 0$$

where θ_1 and θ_2 are the angular displacements of the two pendulums from the vertical. The results of the measurements show that the system exhibits a rich variety of behavior, including periodic motion, quasi-periodic motion, and chaotic motion. The theoretical analysis shows that the system is Hamiltonian and that the equations of motion can be solved exactly. The discussion of the results shows that the experimental results are in good agreement with the theoretical predictions. The conclusion is that the system is a good example of a Hamiltonian system and that the equations of motion can be solved exactly.

CHAPTER 5

DISCUSSION OF RESULTS AND CONCLUSIONS

The contributions which have been made toward the development of "A Microwave, Direction-Modulated Hyperbolic Navigational System" can be best presented by summarizing the results which will be generally applicable, illustrating the application of the general results to a particular situation, and then evaluating the resulting system.

5.1 Summary of General Results

Criteria for selection of station sites have been set forth in Chapter 2. After the stations have been located, the values of required introduced delay can be determined by Equation 5. Equation 12 can be used to determine the system error in terms of the timer error (e_t), the indicator error (e_i), and the error due to the variation in introduced delay per beamwidth (e_d). Equation 8 can be used to obtain data for plotting the direction-modulated hyperbolic pattern.

Geometrical relations in a particular application determine the variation in delay per degree of rotation of the Rotating station antenna. With the type of indicator selected (see Chapter 4), the operator determines the mean of extreme values of time delay received. For a linear variation of d with θ within a beamwidth, the mean value will equal the actual value at the center of the beam. Thus e_d is introduced by non-linearity of d vrs. θ . The value of e_d will be the difference between the mean value and the actual value. Values of e_d can be determined from a plot of d vrs. θ . (see Figure 5.1). Significant non-linearity can be expected where the trackline changes direction, and where the trackline

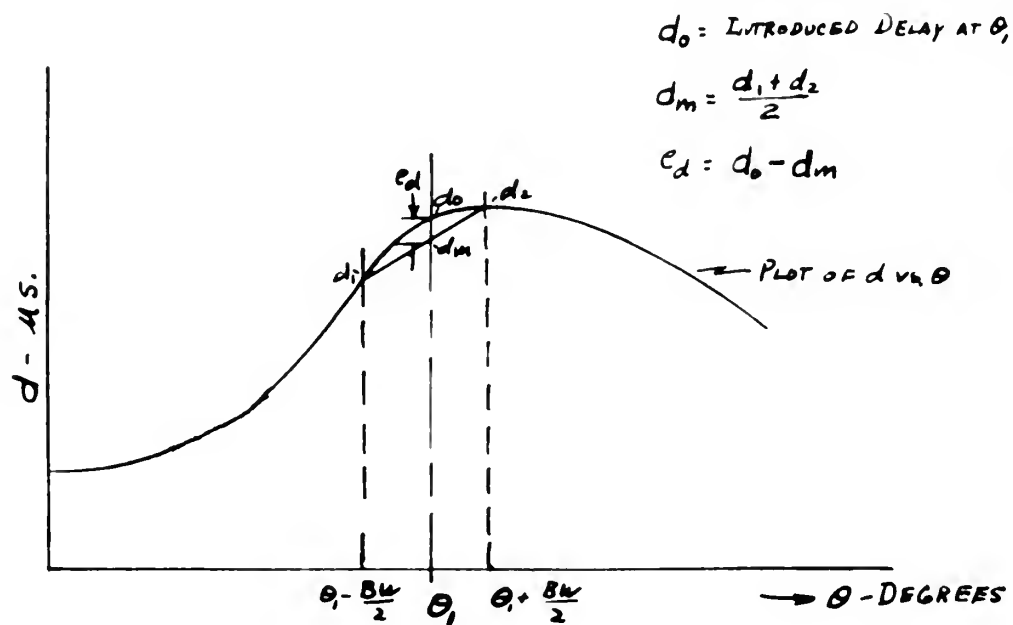


FIGURE 5.1

ILLUSTRATION OF INCIDENCE OF e_d - ERROR
CAUSED BY NON-LINEARITY OF d vs. θ

becomes tangent to the normal hyperbolic pattern.

5.2 Determination of Timer and Indicator Errors

The quantity e_i is the error in reading time delay through the complete system when e_t and e_d are zero. Its value will depend on equipment components, the spread of signals caused by variation of d per degree, the sweplength of the presentation, the rate of antenna rotation, and the beamwidth.

The value of e_i must be determined experimentally. It is convenient and almost necessary to include the timer in the test and determine $(e_i + e_t)$. An experiment similar to the test of R.F. link in Chapter 4 could be used.

The spread in d to be introduced for the experimental test should be the maximum variation in d per beamwidth, as determined by a plot of d vrs. θ . The sweplength of the indicator should be such that for any position in the channel, the signal and entire spread of signals would appear on the trace. From inspection of the plot of the direction-modulated hyperbolic pattern, the maximum value of T within the channel can be determined. The maximum variation of d for half a beamwidth can be determined from a plot of d vrs. θ . The sweplength should be twice the sum of these values. The antenna rotation can be simulated by gating the trigger to one transmitter at a rate equal to the rate of antenna rotation, and with a gate width equal to the time required for the antenna to turn through a beamwidth.

A point by point analysis could be made for e_i based on the actual spread. Due to the rather indefinite dependence, it is considered better to take the value for the maximum spread and consider it as constant.

5.3 Practical Results

The procedure outlined in the previous sections was developed as a result of all theoretical and experimental work done. Consequently it was not followed exactly in applying the system to Boston Harbor; the same essential information was, however, obtained.

The location of stations and determination of geometric relations for Boston Harbor was discussed in Chapter 3. The maximum value of P.D. was $1,334$ ft./us.

To determine the maximum system error, Equation 12 need be evaluated only at the points where P.D. is maximum, and where e_d is significant in the summation of $(e_t + e_i + e_d)$.

By the experimental procedure discussed in Chapter 4, $(e_t + e_i)$ was found to be ± 0.03 us. For Boston Harbor e_d became significant at only one point. It was 0.025 us. where P.D. was 497 ft./us. For this point E is 27.4 ft. Therefore:

$$E_{\max} = (e_t + e_i) \text{ P.D.}_{\max}$$

$$E_{\max} = 0.03 \times 1,334 = 40 \text{ ft.}$$

5.4 Conclusions

5.4.1 Accuracy for Boston Harbor. For quick and easy determination of time delay, it is felt the probable error based on the figure of $.03$ us. for $e_t + e_i$) is a bit optimistic. Reference to Figure A.6.2 shows that a time difference of 0.1 us. is readily apparent. Based on this more conservative figure:

$$E_{\max} = 0.1 \times 1,334 = 133.4 \text{ ft.}$$

With this accuracy the system could be used to permit two-way traffic in a 400 yd. channel. Ships could stay at least 50 yards from the edge

The location of stations and their relations
to Boston Harbor are discussed in Chapter I.
The maximum value of χ , χ_{max} , was 1.33 ft.

It is obvious that the above results are not sufficient to determine the value of α and β and where α is the coefficient of x and β is the coefficient of x^2 in the expansion of $(1+x)^n$.

By the experimental procedure discussed in Chapter 4, $(\sigma_1 + \sigma_2)$ was found to be 1.03 ± 0.03 for carbon having δ between significant at only one point. It was 1.025 ± 0.005 where δ was 12.1 for this point. Therefore:

$$x_{\text{max}}(1, 110 + j0) = x_{\text{max}}$$

2007-01-02 4.2

Fig. 1. Accuracy for Boston Harbor. For high and low deformation of
time delay, it is felt the probable error based on the figure of 0.1 us.
for $\sigma_1 + \sigma_2$ is a little optimistic. Reference to Figure 1.2 shows this
a time difference of 0.1 us. is readily apparent. Based on this more
conservative estimate

$$1.91 \times 10^3 = 100.1 \times 1.0 = 100.1$$

in a 1960 car, license 1-234567. Since this car is used to make trips to the north and south of the city, it is not possible to determine the exact date of purchase.

of the channel, and passing ships would be separated by at least 100 yards.

5.4.2 Extension of Application. In considering extension of the application of this system, several points should be mentioned.

The error e_d can be ignored in determination of maximum system error on the basis that transmission could be stopped for the few narrow sectors where it would be of consequence.

The beamwidth is not as critical as it originally appeared to be. Its effect on e_d has been discounted, and the figure of 0.03 us. for $(e_t + e_i)$ was obtained with a spread of signals of about 0.5 us., which would be the maximum for Boston with a one degree beamwidth.

The effects of both spread and sampled data were small compared to the increase in $(e_t + e_i)$ with the introduction of the R.F. link. It therefore appears that the equipment was the primary factor in determining the value of 0.03 us. for $(e_t + e_i)$. Equipment specifically designed for use in this system could probably decrease this error.

Ease of rapid determination of time delay (consideration of which dictated increasing the time error to 0.1 us.) will be almost directly proportional to sweep length.

For any installation where stations can be located to keep the desired service area within the baseline circle; a 4 us. sweep would be sufficient, the maximum P.D. would be 964 ft./us., and system error could be limited to less than 100 ft. This would permit safe two-way traffic in a 400 yard channel.

5.4.3 Suggested Improvements. Unfortunately the time delay reading will not be proportional to distance off the trackline throughout the service area, because of variations in P.D. The system could be modified to

The proposed system is designed to provide a means of determining the location of a vehicle in a road network. The system consists of a central computer and a number of roadside units. The roadside units are located at various points along the road network and are capable of receiving signals from a vehicle. The central computer is capable of processing these signals and determining the location of the vehicle. The system is designed to be used in a variety of applications, including traffic management and emergency services.

indicate the two edges of the channel, and safety of position could be more readily determined.

Another possible improvement would be to alter the signal from the Rotating station at several points in the rotation; this could provide indication of progress along the channel.

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CHAPTER 6

RECOMMENDATIONS FOR FUTURE DEVELOPMENT

In development of the system to this point, it has been assumed that the time τ can be controlled by antenna rotation to provide the proper variation of d with θ . Since the timer error was found experimentally to be negligible, this assumption seems justified. However, design of a delay control mechanism is essential to the operation of an actual system. This design problem was not studied. One type of mechanism might be a cam on the antenna shaft with ordinates proportional to required delay at each angle. The cam follower would drive a linear time delay potentiometer in a circuit similar to that of Figure A.3. Other mechanisms might be devised using electronic means only. The details of design for this component should be studied in future development.

Experimental results indicate that, if a control mechanism can be developed, a pilot installation is warranted to test the system under actual operating conditions.

The nature of the signals used in the system suggests the possibility of automatic presentation of position by a meter indicating "on channel", or "left" or "right" of channel. An extension of this type of presentation is readily seen in application of the meter signal to an automatic steering device.

[illegible]

APPENDIX

APPENDIX

A.1 Summary of Symbols and Equations

| | |
|------------------------------|--|
| a..... | Nondimensional distance r/L |
| Baseline..... | Line between transmitting stations |
| BASILINE CIRCLE | Circle drawn with the baseline as a diameter |
| B.W. | Beamwidth |
| c..... | Velocity of light |
| d..... | Introduced delay |
| E..... | System probable error in feet |
| e _d | Error between indicated time difference and actual time difference, in microseconds. |
| e _i | Probable error in reading indicated time difference, in microseconds |
| e _t | Probable timer error in microseconds |
| F..... | Fixed station |
| L..... | Baseline length |
| P.D. | Pattern Divergence |
| R..... | Rotating station |
| r..... | Distance from the Rotating station to a point |
| R.D. | Radial Divergence |
| R.F. Link..... | Inclusion of transmitter and receiver |
| T..... | Time difference |
| Track angle..... | Acute angle between trackline and radius from R |
| us. | Microsecond(s) |
| θ..... | Angle between the baseline and a radius from R to a point |
| φ..... | Track angle |
| ∂..... | Partial derivative |

$$(5).....d = \frac{L(1 - a \sqrt{1 - 2a \cos\theta + a^2})}{c}$$

$$(7).....c \frac{\partial T}{\partial r} \equiv R.S. = \frac{a - \cos\theta}{\sqrt{1 - 2a \cos\theta + a^2}} - 1$$

$$(8).....R.D. \equiv \frac{c}{R.S.}$$

$$(9).....P.D. = R.D. \sin \phi$$

$$(12).....E = (e_d + e_t + e_i)P.D.$$

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1. Loci of Radial Sensitivity

Figure 2.3 showing the loci of various values of Radial Sensitivity was based on the following manipulation of Equation 7, which stated:

$$R.S. = \frac{a - \cos\theta}{\sqrt{1 - 2a \cos\theta + a^2}} - 1$$

Rearrange with $(a - \cos\theta)$ on the right side, and square both sides:

$$(R.S.^2 + 2R.S. + 1)(1 - 2a \cos\theta + a^2) = a^2 - 2a \cos\theta + \cos^2\theta$$

$$(R.S.^2 + 2R.S.)a^2 - 2\cos\theta(R.S.^2 + 2R.S.)a + (R.S. + 1)^2 - \cos^2\theta = 0$$

$$a = \cos\theta + \frac{\sqrt{\cos^2\theta - (R.S. + 1)^2 - \cos\theta}}{(R.S.^2 + 2R.S.)}$$

$$a = \cos\theta + \frac{\sqrt{(R.S. + 1)^2(\cos^2\theta - 1)}}{R.S.^2 + 2R.S.}$$

$$a = \cos\theta + \frac{R.S. + 1}{\sqrt{-(R.S.^2 + 2R.S.)}} \sqrt{1 - \cos^2\theta}$$

$$a = \cos\theta + \frac{R.S. + 1}{\sqrt{-(R.S.)^2 - 2R.S.}} \sin\theta$$

$$\text{Let } \frac{R.S. + 1}{\sqrt{-(R.S.)^2 - 2R.S.}} = K, \text{ then}$$

$$a = \cos\theta + K \sin\theta.$$

Converting to rectangular co-ordinates,

$$\frac{x}{\sqrt{x^2 + y^2}} + \frac{Ky}{\sqrt{x^2 + y^2}} = 1$$

Let α be a root of $x^2 - 1 = 0$ in K .

Then $\alpha^2 = 1$ and $\alpha \neq 1$ (since $\alpha \neq 1$).

$$\alpha + \alpha^{-1} = \alpha + \alpha = 2\alpha$$

$$\alpha^2 + \alpha^{-2} = 1 + 1 = 2$$

Let β be a root of $x^2 - 2\alpha x + 1 = 0$ in $K(\alpha)$.

$$\beta^2 - 2\alpha\beta + 1 = 0 \implies \beta^2 + 1 = 2\alpha\beta$$

$$\beta^4 + 1 = 2\alpha\beta(\beta^2 + 1) = 2\alpha\beta(2\alpha\beta) = 4\alpha^2\beta^2 = 4\beta^2$$

$$\beta^4 + 1 = 4\beta^2 \implies \beta^4 - 4\beta^2 + 1 = 0$$

$$\beta^4 - 4\beta^2 + 1 = 0 \implies \beta^2 = \frac{4 \pm \sqrt{16 - 4}}{2} = 2 \pm \sqrt{3}$$

$$\beta^2 = 2 + \sqrt{3} \implies \beta = \pm \sqrt{2 + \sqrt{3}}$$

$$\beta^2 = 2 - \sqrt{3} \implies \beta = \pm \sqrt{2 - \sqrt{3}}$$

$$\beta = \pm \sqrt{2 + \sqrt{3}} \text{ or } \pm \sqrt{2 - \sqrt{3}}$$

$$\beta = \pm \sqrt{2 + \sqrt{3}}$$

$$\beta = \pm \sqrt{2 + \sqrt{3}}$$

$$\beta = \pm \sqrt{2 + \sqrt{3}}$$

$$\beta = \pm \sqrt{2 + \sqrt{3}}$$

Converting to rectangular coordinates

$$\beta = \sqrt{2 + \sqrt{3}} = \sqrt{2 + \sqrt{3}} \cdot \frac{\sqrt{2 - \sqrt{3}}}{\sqrt{2 - \sqrt{3}}} = \frac{\sqrt{(2 + \sqrt{3})(2 - \sqrt{3})}}{\sqrt{2 - \sqrt{3}}} = \frac{\sqrt{4 - 3}}{\sqrt{2 - \sqrt{3}}} = \frac{1}{\sqrt{2 - \sqrt{3}}}$$

$$x^2 + y^2 = x + K$$

$$x^2 + y^2 - x - Ky = 0$$

This is a standard form of a circle⁴ with center at $(\frac{1}{2}, \frac{K}{2})$ and radius of $\frac{1}{2} \sqrt{1 + K^2}$. The distance from the center to the origin will be $\frac{1}{2} \sqrt{1 + K^2}$ so the circle passes through the origin.

Translated back to polar co-ordinates, as defined originally in Figure 2.2, the loci will be a circle passing through both stations, with center on the perpendicular bisector of the baseline a distance $\frac{KL}{2}$ from the baseline.

Note that for R.S. = -1, $K = 0$ so the locus will be a circle with the baseline as a diameter. (This is substantiated by letting R.S. = -1 in Equation 7. The result is $a = \cos\theta$.) Continuous use of positive values of square roots may be corrected for with the following stipulations:

1. For values of R.S. numerically greater than one the locus will be that part of the locus circle within the baseline circle.
2. For values of R.S. numerically less than one the locus will be that part of the locus circle outside the baseline circle.

TABLE II

| <u>R.S.</u> | <u>K/2</u> |
|-------------|------------|
| -2 | ∞ |
| -1.5 | -.288 |
| -1.0 | 0 |
| -0.5 | +.288 |
| -0.25 | +.568 |

The sample provided as evidence was analyzed by the FBI Laboratory and found to contain traces of lead, iron, and copper. The results of the analysis are being compared with those obtained from other samples.

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1. *Chlorophyll a* and *Chlorophyll b* were determined by the method of Lichtenthaler and Whistler (1973).

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Figure 1

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A.3 Derivation of Equation for e_d

Under certain conditions e_d is equal to one-half the variation in d per beamwidth, or

$$e_d = \frac{d(d)}{d\theta} \frac{B.W.}{2} \quad (10)$$

Equation

$$d = \frac{L(1 - a + \sqrt{1 - 2a \cos\theta + a^2})}{c}$$

is valid for points on the trackline, so a must be a function of θ .

Therefore:

$$\frac{d(d)}{d\theta} = \frac{\partial d}{\partial \theta} + \frac{\partial d}{\partial a} \frac{da}{d\theta} \quad (14)$$

$$\frac{\partial d}{\partial \theta} = \frac{aL}{c} \frac{\sin\theta}{\sqrt{1 - 2a \cos\theta + a^2}}$$

$$\frac{\partial d}{\partial a} = \frac{L}{c} \left(-1 + \frac{-\cos\theta + a}{\sqrt{1 - 2a \cos\theta + a^2}} \right)$$

From Figure A-1, it can be seen that

$$\frac{da}{d\theta} = a \cot\phi$$

Combining the above as per Equations 10 and 14,

$$e_d = \frac{aL}{c} \left\{ \frac{\sin\theta}{\sqrt{1 - 2a \cos\theta + a^2}} + \left(\frac{a - \cos\theta}{\sqrt{1 - 2a \cos\theta + a^2}} - 1 \right) \cot\phi \right\} \frac{B.W.}{2} \quad (11)$$

(12)

λ_{12}

1917, 17

$$\lambda_{12} + \lambda_{21} = \lambda_{11} + \lambda_{22} = 1$$

15 In 1917

the 1917-18 season was a very dry one and the yield of

the 1917-18 season

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$$\lambda_{12} + \lambda_{21} = \lambda_{11} + \lambda_{22} = 1$$

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$$\lambda_{12} + \lambda_{21} = \lambda_{11} + \lambda_{22} = 1$$

the 1917-18 season was a very dry one and the yield of

$$\lambda_{12} + \lambda_{21} = \lambda_{11} + \lambda_{22} = 1$$

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$$\lambda_{12} + \lambda_{21} = \lambda_{11} + \lambda_{22} = 1$$

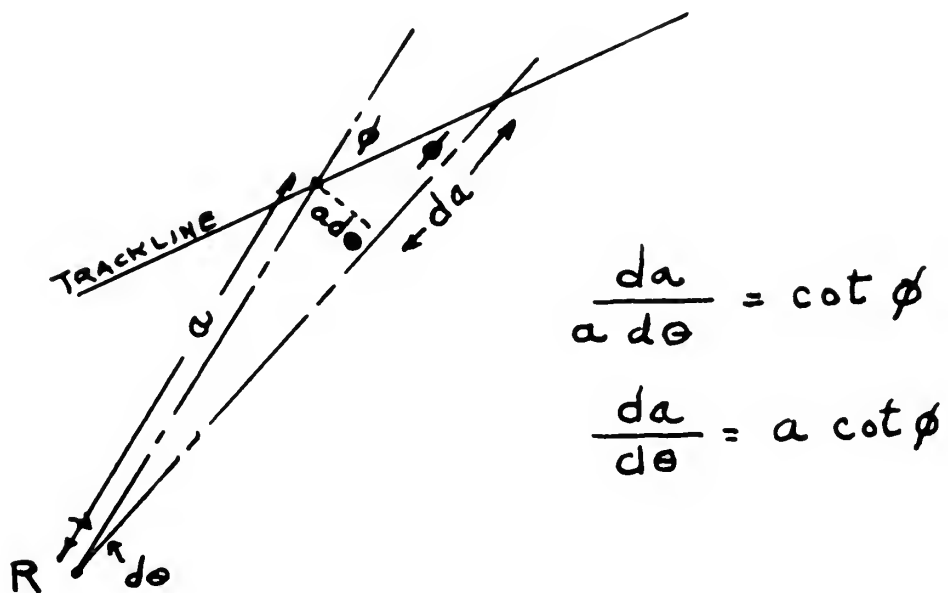


FIGURE A.1
DETERMINATION OF $da/d\theta$



A.4 Detail Data for Boston

Table III

| θ | aL | a | $\cos\theta$ | $\sqrt{1-2a \cos\theta + a^2}$ | d | $d/\text{deg.}$ | R.S. | R.D. | ϕ | P.D. |
|----------|-------|------|--------------|--------------------------------|-------|-----------------|--------|--------|--------|--------|
| -80 | 7.90 | .535 | .174 | 1.050 | 47.3 | .20 | -.658 | 1497 | 60° | 1293 |
| -75 | 7.56 | .520 | .256 | 1.002 | 46.3 | .19 | -.736 | 1336 | 66 | 1220 |
| -70 | 7.31 | .496 | .342 | .952 | 45.4 | .18 | -.838 | 1172 | 70 | 1100 |
| -65 | 7.22 | .480 | .423 | .908 | 44.5 | .28 | -.937 | 1050 | 82 | 1038 |
| -60 | 7.36 | .500 | .500 | .866 | 42.6 | .35 | - 1 | 984 | 77 | 959 |
| -55 | 7.52 | .510 | .574 | .822 | 41.0 | .37 | -1.078 | 914 | 70.5 | 861 |
| -50 | 7.81 | .530 | .643 | .775 | 38.9 | .44 | -1.146 | 851 | 67 | 790 |
| -45 | 8.15 | .553 | .707 | .724 | 36.6 | .50 | -1.213 | 811 | 61 | 708 |
| -40 | 8.59 | .582 | .766 | .669 | 33.9 | .50 | -1.275 | 771 | 56 | 639 |
| -35 | 8.89 | .602 | .819 | .614 | 31.6 | .50 | -1.354 | 726 | 69 | 677 |
| -30 | 9.25 | .627 | .866 | .552 | 28.9 | .51 | -1.433 | 686 | 67 | 632 |
| -25 | 9.55 | .648 | .906 | .497 | 26.5 | .43 | -1.519 | 648 | 84 | 645 |
| -20 | 9.67 | .656 | .940 | .445 | 24.6 | .37 | -1.638 | 601 | 81 | 594 |
| -15 | 9.80 | .665 | .966 | .395 | 22.8 | .78 | -1.762 | 558 | 88 | 556 |
| -10 | 9.80 | .665 | .985 | .363 | 21.8 | .19 | -1.882 | 522 | 87 | 521 |
| - 5 | 9.88 | .670 | .996 | .338 | 20.9 | .19 | -1.965 | 500 | 84 | 498 |
| 0 | 10.01 | .680 | 1 | .319 | 19.90 | .18 | -2.00 | 492 | 78 | 482 |
| 5 | 10.26 | .696 | .996 | .303 | 19.1 | .2 | -1.975 | 498 | 87 | 497 |
| 10 | 10.25 | .695 | .985 | .338 | 20.1 | .25 | -1.857 | 530 | 89 | 529 |
| 15 | 10.12 | .686 | .966 | .379 | 21.6 | .37 | -1.738 | 566 | 77 | 552 |
| 20 | 9.92 | .673 | .940 | .434 | 23.8 | .41 | -1.615 | 609 | 80 | 600 |
| 25 | 9.81 | .665 | .906 | .487 | 25.7 | .35 | -1.494 | 658 | 86 | 657 |
| 30 | 9.82 | .666 | .866 | .540 | 27.3 | .31 | -1.371 | 716 | 89 | 716 |
| 35 | 9.93 | .673 | .819 | .594 | 28.8 | .18 | -1.246 | 789 | 76 | 764 |
| 40 | 10.20 | .691 | .766 | .623 | 29.1 | .22 | -1.120 | 879 | 69 | 820 |
| 45 | 10.56 | .716 | .707 | .707 | 31.0 | .32 | -.987 | 997 | 64 | 895 |
| 50 | 10.86 | .736 | .643 | .772 | 32.3 | .33 | -.880 | 1118 | 80 | 1100 |
| 55 | 10.85 | .736 | .574 | .835 | 34.3 | .41 | -.806 | 1220 | 89 | 1220 |
| 60 | 10.84 | .735 | .500 | .898 | 36.4 | .42 | -.738 | 1334 | 90 | 1334 |
| | | | | | us/dg | ft/dg. | | ft/us. | | ft/us. |

$L = 14.75 \text{ in (on chart)} = 30.729 \text{ ft.}$

$$d/\text{deg} = \frac{d(\theta + 5) - d(\theta - 5)}{10}$$

$c = 984 \text{ ft/us.}$

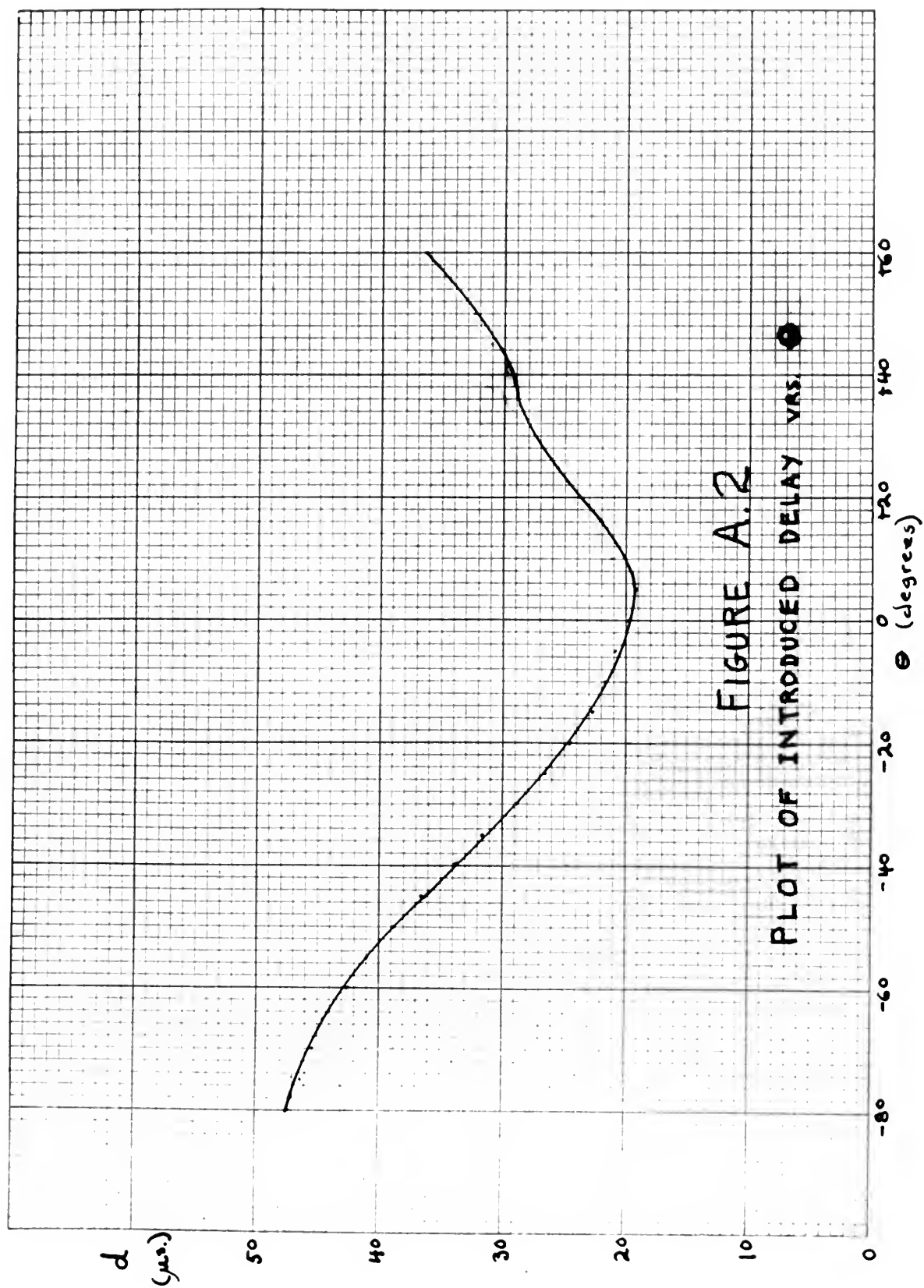


FIGURE A.2

PLOT OF INTRODUCED DELAY VRS. θ

4. Details of Experiments

This section will include a description of equipment used in, and raw data obtained. Procedures followed to obtain data, and analysis of data to arrive at conclusions is in the body of the report.

Test of Critical Components Without the R.F. Link

Data, items 1-5, were obtained to determine timer-indicator accuracy under ideal conditions, to calibrate the variable time delay, and to determine best indicator pattern for presentation. Items 6-11 were obtained to determine effects of various factors on accuracy.

Test of Critical Components With the R.F. Link

Data, items 1, 2 were obtained to find accuracy of position determination by the system. Items 3 were obtained to determine overall errors of system with the R.F. Link.

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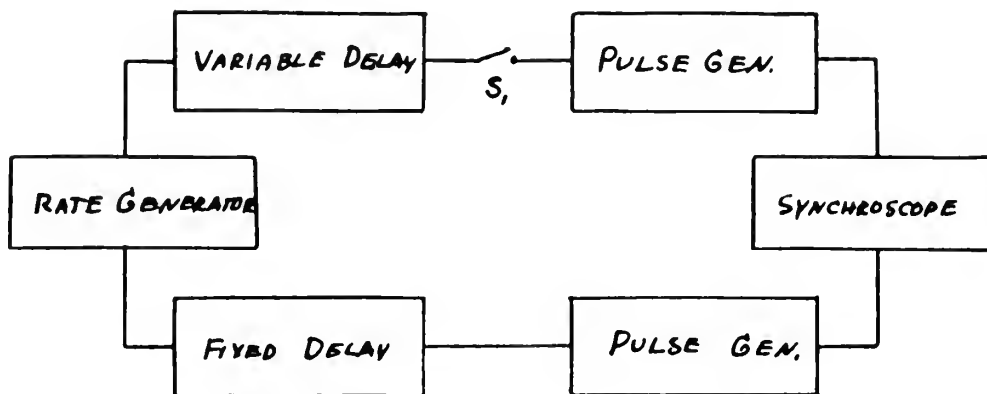


FIGURE A. 4

DIAGRAM OF EQUIPMENT USED FOR TEST
No. 1 (WITHOUT R.F. LINK)

RATE GEN. AND SYNCHROSCOPE - TEXTRON, TYPE 513D

PULSE GENERATORS - HEWLETT PACKARD, MOD. 212A

FIXED DELAY - BUILT INTO PULSE GENERATORS

VARIABLE DELAY - CONSTRUCTED FROM DIAGRAM, FIG. A.3

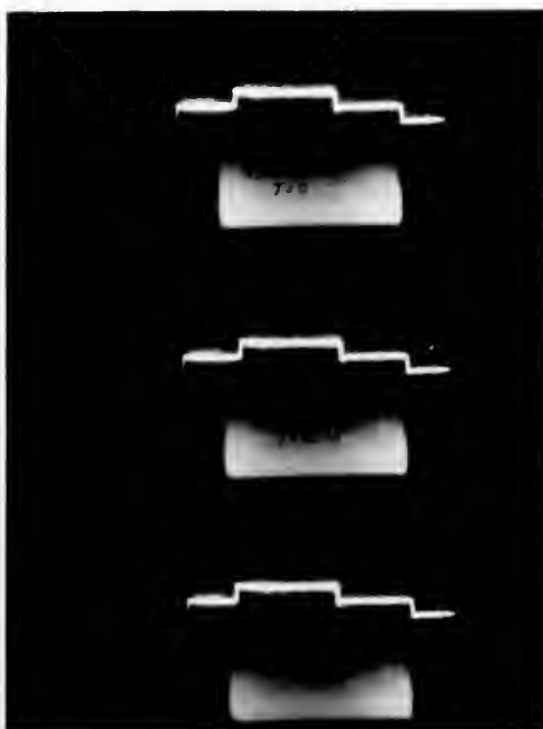


Fig. A.5.1



Fig. A.5.2

FIGURE A.5 Types of Indicator patterns.

Figure A.5.1 shows the type of pattern presented by superposition of pulses from the pulse generators. The first pulse received triggers the sweep, and also appears on the sweep. Pulse lengths were 2 us. and 4 us. respectively. Conditions shown are for $T = 0$, $T = 0.1$ us., and $T = 0.2$ us.

Figure A.5.2 shows the pattern when the sweep is triggered by the first pulse, and the second pulse only appears on the sweep. Time delay conditions are the same as for Fig. A.5.1

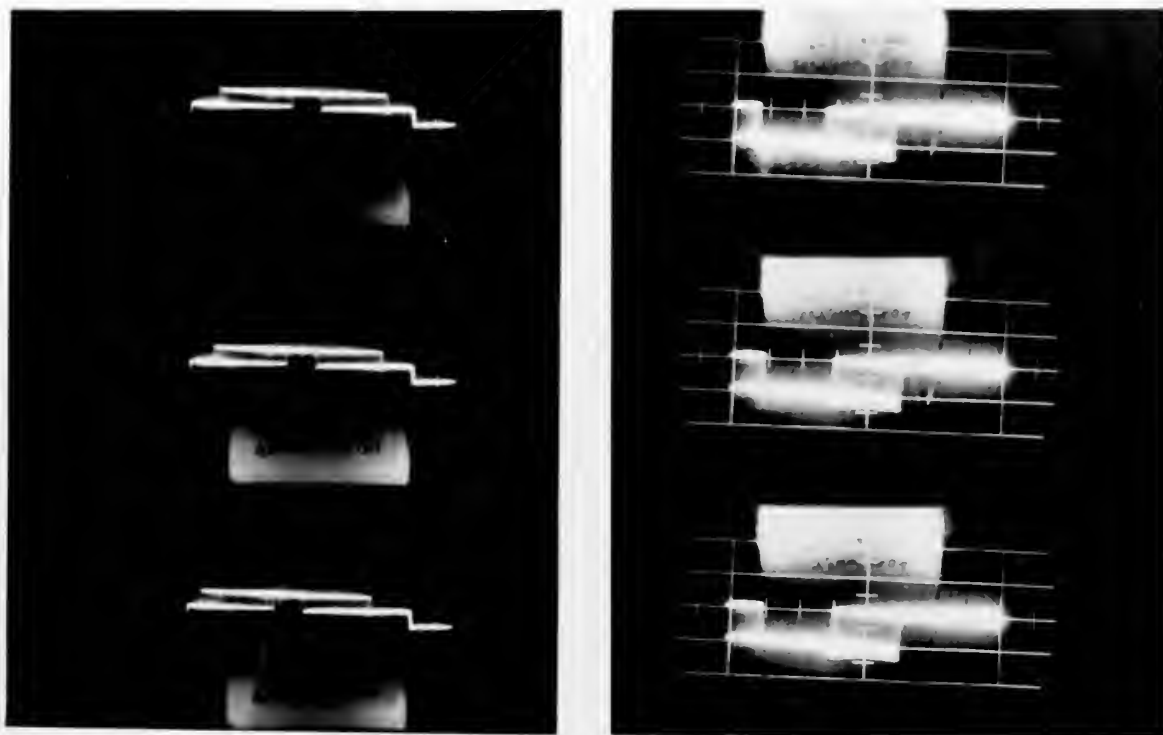


FIGURE A.6

COMPARISON OF INDICATOR PATTERNS WITH SPREAD

In each picture above, the value of T is shown at zero, 0.1 us., and 0.2 us. In both types of presentation, it is readily apparent that a mismatch of 0.1 us. can be seen at a glance. With care this accuracy might be improved. These observations are highly significant, since they confirm the conclusions stated, that probable overall system error will be 0.1 us.



Figure A.7.1

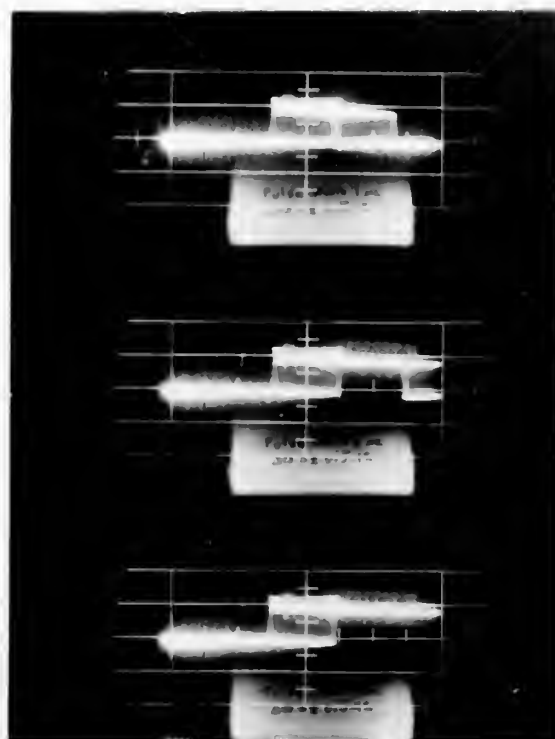


Figure A.7.2

FIGURE A.7

Effects of Pulse Length and Spread

Figure A.7.1 shows the picture obtained with a spread of 0.5 us., 0.2 us., and 0.1 us. Figure A.7.2 shows the effect of pulse length on the Step-Type of indication. Here it is apparent that the pulse length of the Rotating Station signal should be as long as the maximum spread plus the maximum channel width in us. in order to avoid ambiguity in interpretation of the received signal. Pulse lengths of 1 us., 2 us., and 4 us. are shown with a constant spread of 0.5 us.

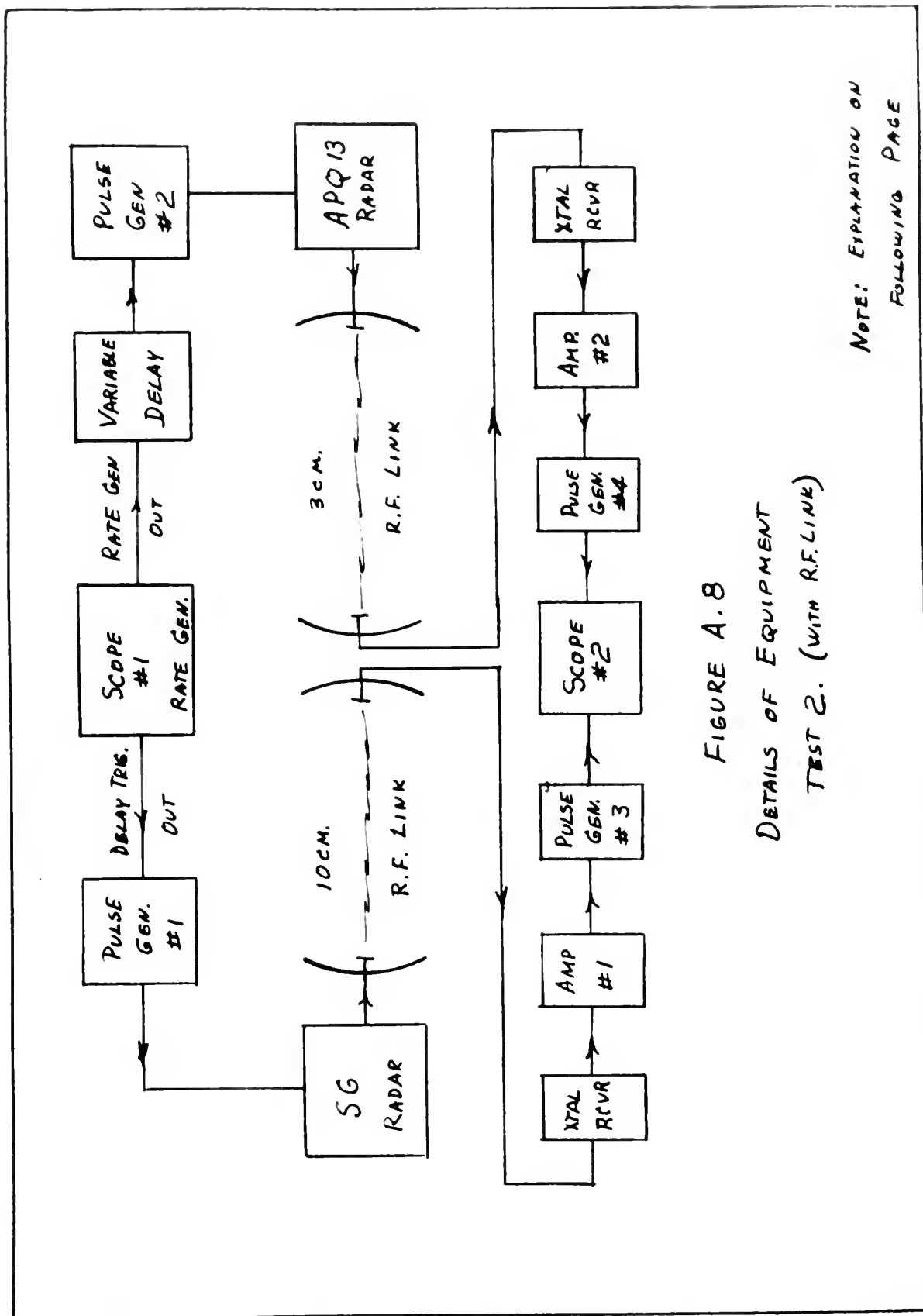


FIGURE A.8

DETAILS OF EQUIPMENT
TEST 2. (WITH R.F. LINK)

NOTE: EXPLANATION ON
FOLLOWING PAGE

IDENTIFICATION OF FIGURE A.3

Scope #1.....Textron, Type 513B

Scope #2.....TS - 34/AP (on 5 us. sweep)

Pulse Gen. #1....Meas. Corp. Model 798

Pulse Gen. #2....Gen. Radio, Type 869A

Pulse Gen. #3....Hewlett Packard, Model 212A

Pulse Gen. #4....Hewlett Packard, Model 212A

Amp. #1.....Hewlett Packard, Model 4000 (60db gain)

Amp. #2.....Hewlett Packard, Model 4000 (60db gain)

Note #1.....Trigger taken to pins 3 & 6 of V-401 in SG Modulation

Gen. Unit (with V-401 removed)

Note #2.....Trigger taken to terminal #16 on modulator unit of APQ 13.

Note #3.....Voltage probe placed in SG waveguide. Cable leads to
connector in coaxial feed

Note 25.....Holtz, George, 1900-1901, 1902-1903, 1904-1905, 1906-1907, 1908-1909, 1910-1911, 1912-1913, 1914-1915, 1916-1917, 1918-1919, 1920-1921, 1922-1923, 1924-1925, 1926-1927, 1928-1929, 1930-1931, 1932-1933, 1934-1935, 1936-1937, 1938-1939, 1940-1941, 1942-1943, 1944-1945, 1946-1947, 1948-1949, 1950-1951, 1952-1953, 1954-1955, 1956-1957, 1958-1959, 1960-1961, 1962-1963, 1964-1965, 1966-1967, 1968-1969, 1970-1971, 1972-1973, 1974-1975, 1976-1977, 1978-1979, 1980-1981, 1982-1983, 1984-1985, 1986-1987, 1988-1989, 1990-1991, 1992-1993, 1994-1995, 1996-1997, 1998-1999, 2000-2001, 2002-2003, 2004-2005, 2006-2007, 2008-2009, 2010-2011, 2012-2013, 2014-2015, 2016-2017, 2018-2019, 2020-2021, 2022-2023, 2024-2025, 2026-2027, 2028-2029, 2030-2031, 2032-2033, 2034-2035, 2036-2037, 2038-2039, 2040-2041, 2042-2043, 2044-2045, 2046-2047, 2048-2049, 2050-2051, 2052-2053, 2054-2055, 2056-2057, 2058-2059, 2060-2061, 2062-2063, 2064-2065, 2066-2067, 2068-2069, 2070-2071, 2072-2073, 2074-2075, 2076-2077, 2078-2079, 2080-2081, 2082-2083, 2084-2085, 2086-2087, 2088-2089, 2090-2091, 2092-2093, 2094-2095, 2096-2097, 2098-2099, 2100-2101, 2102-2103, 2104-2105, 2106-2107, 2108-2109, 2110-2111, 2112-2113, 2114-2115, 2116-2117, 2118-2119, 2120-2121, 2122-2123, 2124-2125, 2126-2127, 2128-2129, 2130-2131, 2132-2133, 2134-2135, 2136-2137, 2138-2139, 2140-2141, 2142-2143, 2144-2145, 2146-2147, 2148-2149, 2150-2151, 2152-2153, 2154-2155, 2156-2157, 2158-2159, 2160-2161, 2162-2163, 2164-2165, 2166-2167, 2168-2169, 2170-2171, 2172-2173, 2174-2175, 2176-2177, 2178-2179, 2180-2181, 2182-2183, 2184-2185, 2186-2187, 2188-2189, 2190-2191, 2192-2193, 2194-2195, 2196-2197, 2198-2199, 2200-2201, 2202-2203, 2204-2205, 2206-2207, 2208-2209, 2210-2211, 2212-2213, 2214-2215, 2216-2217, 2218-2219, 2220-2221, 2222-2223, 2224-2225, 2226-2227, 2228-2229, 2230-2231, 2232-2233, 2234-2235, 2236-2237, 2238-2239, 2240-2241, 2242-2243, 2244-2245, 2246-2247, 2248-2249, 2250-2251, 2252-2253, 2254-2255, 2256-2257, 2258-2259, 2260-2261, 2262-2263, 2264-2265, 2266-2267, 2268-2269, 2270-2271, 2272-2273, 2274-2275, 2276-2277, 2278-2279, 2280-2281, 2282-2283, 2284-2285, 2286-2287, 2288-2289, 2290-2291, 2292-2293, 2294-2295, 2296-2297, 2298-2299, 2300-2301, 2302-2303, 2304-2305, 2306-2307, 2308-2309, 2310-2311, 2312-2313, 2314-2315, 2316-2317, 2318-2319, 2320-2321, 2322-2323, 2324-2325, 2326-2327, 2328-2329, 2330-2331, 2332-2333, 2334-2335, 2336-2337, 2338-2339, 2340-2341, 2342-2343, 2344-2345, 2346-2347, 2348-2349, 2350-2351, 2352-2353, 2354-2355, 2356-2357, 2358-2359, 2360-2361, 2362-2363, 2364-2365, 2366-2367, 2368-2369, 2370-2371, 2372-2373, 2374-2375, 2376-2377, 2378-2379, 2380-2381, 2382-2383, 2384-2385, 2386-2387, 2388-2389, 2390-2391, 2392-2393, 2394-2395, 2396-2397, 2398-2399, 2400-2401, 2402-2403, 2404-2405, 2406-2407, 2408-2409, 2410-2411, 2412-2413, 2414-2415, 2416-2417, 2418-2419, 2420-2421, 2422-2423, 2424-2425, 2426-2427, 2428-2429, 2430-2431, 2432-2433, 2434-2435, 2436-2437, 2438-2439, 2440-2441, 2442-2443, 2444-2445, 2446-2447, 2448-2449, 2450-2451, 2452-2453, 2454-2455, 2456-2457, 2458-2459, 2460-2461, 2462-2463, 2464-2465, 2466-2467, 2468-2469, 2470-2471, 2472-2473, 2474-2475, 2476-2477, 2478-2479, 2480-2481, 2482-2483, 2484-2485, 2486-2487, 2488-2489, 2490-2491, 2492-2493, 2494-2495, 2496-2497, 2498-2499, 2500-2501, 2502-2503, 2504-2505, 2506-2507, 2508-2509, 2510-2511, 2512-2513, 2514-2515, 2516-2517, 2518-2519, 2520-2521, 2522-2523, 2524-2525, 2526-2527, 2528-2529, 2530-2531, 2532-2533, 2534-2535, 2536-2537, 2538-2539, 2540-2541, 2542-2543, 2544-2545, 2546-2547, 2548-2549, 2550-2551, 2552-2553, 2554-2555, 2556-2557, 2558-2559, 2560-2561, 2562-2563, 2564-2565, 2566-2567, 2568-2569, 2570-2571, 2572-2573, 2574-2575, 2576-2577, 2578-2579, 2580-2581, 2582-2583, 2584-2585, 2586-2587, 2588-2589, 2590-2591, 2592-2593, 2594-2595, 2596-2597, 2598-2599, 2600-2601, 2602-2603, 2604-2605, 2606-2607, 2608-2609, 2610-2611, 2612-2613, 2614-2615, 2616-2617, 2618-2619, 2620-2621, 2622-2623, 2624-2625, 2626-2627, 2628-2629, 2630-2631, 2632-2633, 2634-2635, 2636-2637, 2638-2639, 2640-2641,

CONDENSED PIV. RAN TEST DATA - WITHOUT A.F. LINE

Continuous signals, no spread, F.R.F. = 2,000 per sec.
Constant delays were set on "fixed" delay and matched by rotation of
Variable Delay Potentiometer

1. Pulses from Fixed and Rotating Pulse Gen. Same Amplitude & Same Width

| Test | A | B | C | D | |
|----------|----------|----------|-------|----------|---|
| Settings | 137 | 143 | 144 | 140 | Width and Height of
Pulse Resulting from
superposed pulse
were maximized |
| | 136 | 141 | 143 | 142 | |
| | 138 | 143 | 142 | 141 | |
| | <u>—</u> | <u>—</u> | (136) | <u>—</u> | |

Probable Error $0.54^\circ = 0.0054 \text{ us.}$

2. Pulse Widths Ratio = 1; amplitude Ratio = 2

| Test | A | B | |
|----------|------------|------------|--|
| Settings | 137 | 137 | Height of resultant
pulse maximized |
| | 137 | 138 | |
| | 137 | 137 | |
| | <u>137</u> | <u>138</u> | |

Probable Error $0.384^\circ = 0.0384 \text{ us.}$

3. Pulse width ratio = 2; amplitude Ratio = 1

| Test | A | B | C | D | |
|----------|------------|-----------|------------|--------------|---------------------------------|
| Settings | 102 | 99 | 102 | 102 | Pulse centers are
superposed |
| | 102 | 100 | 101 | 102 | |
| | 102 | 99 | 100 | 102 | |
| | <u>101</u> | <u>99</u> | <u>101</u> | <u>102.5</u> | |

Probable Error $0.54 = 0.0054 \text{ us.}$

4. Pulse width Ratio = 1.1; Amplitude Ratio = 1

| Test | A | B | |
|----------|--------------|--------------|-----------------------------|
| Settings | 117.5 | 117 | Pulse centers
superposed |
| | 118 | 115 | |
| | 117 | 117 | |
| | <u>117.5</u> | <u>117.5</u> | |

Probable Error $0.384^\circ = 0.00384 \text{ us.}$

... ..

... ..

| Test | | | |
|---------------------------|-----|-----|-----|
| Settings | 100 | 100 | 100 |
| | 100 | 100 | 100 |
| | 100 | 100 | 100 |
| | 100 | 100 | 100 |
| Probable error 0.0004 ms. | | | |

2. False ratio = 1; amplification ratio = 1

| Test | | | |
|---------------------------|-----|-----|-----|
| Settings | 100 | 100 | 100 |
| | 100 | 100 | 100 |
| | 100 | 100 | 100 |
| | 100 | 100 | 100 |
| Probable error 0.0004 ms. | | | |

3. False ratio = 1; amplification ratio = 1

| Test | | | |
|---------------------------|-----|-----|-----|
| Settings | 100 | 100 | 100 |
| | 100 | 100 | 100 |
| | 100 | 100 | 100 |
| | 100 | 100 | 100 |
| Probable error 0.0004 ms. | | | |

4. False ratio = 1; amplification ratio = 1

| Test | | | |
|---------------------------|-----|-----|-----|
| Settings | 100 | 100 | 100 |
| | 100 | 100 | 100 |
| | 100 | 100 | 100 |
| | 100 | 100 | 100 |
| Probable error 0.0004 ms. | | | |

5. Pulse width Ratio = 1.1; Amplitude Ratio = 2

| Test | A | B | C | D | |
|----------|--------------|--------------|--------------|--------------|-----------------------------|
| | 116.5 | 117.0 | 114.0 | 114.0 | Pulse Centers
superposed |
| Settings | 116.0 | 113.5 | 115.0 | 114.0 | |
| | 116.0 | 114.5 | 114.0 | 114.0 | |
| | <u>116.0</u> | <u>114.0</u> | <u>114.0</u> | <u>114.0</u> | |

Probable Error $0.18^\circ = 0.0018 \text{ us.}$ $0.334^\circ = 0.00334 \text{ us}$

The Following tests were Conducted with P.R.R. = 5,000/sec.

6. Pulse width Ratio = 2; Amplitude Ratio = 1, Spread = 0.75 us.

| Test | A | B | C | |
|----------|-----------|-----------|-----------|-----------------------------|
| | 20 | 26 | 25 | Pulse centers
Superposed |
| Settings | 23 | 23 | 30 | |
| | 20 | 26 | 28 | |
| | 26 | 20 | 28 | |
| | <u>20</u> | <u>25</u> | <u>27</u> | |

Probable Error $1.3^\circ = 0.013 \text{ us.}$ C - Intermittent Signals 2/sec or 120/min.

7. Step Type Indication: No Spread; continuous Signals

| Test | A | B | |
|------|------------|------------|---------------------------|
| | 195 | 195 | Step Centered on
Sweep |
| | 195 | 196 | |
| | 194 | 196 | |
| | 195 | 197 | |
| | <u>196</u> | <u>196</u> | |

Probable Error $0.68^\circ = 0.0068^\circ$

8. Step Type Indication: Spread = 0.5 us.; Continuous Signals

| Test | A | B |
|----------|------------|------------|
| | 169 | 170 |
| Settings | 170 | 171 |
| | 172 | 172 |
| | 172 | 170 |
| | <u>172</u> | <u>170</u> |

Probable Error $0.8^\circ = 0.008 \text{ us.}$

| | | | | |
|----|----|----|----|----|
| 10 | 11 | 12 | 13 | 14 |
| 15 | 16 | 17 | 18 | 19 |
| 20 | 21 | 22 | 23 | 24 |
| 25 | 26 | 27 | 28 | 29 |
| 30 | 31 | 32 | 33 | 34 |

The following table shows the results of the tests.

| Test | 1 | 2 | 3 |
|------------|-----|-----|-----|
| 1st test | 10 | 11 | 12 |
| 2nd test | 13 | 14 | 15 |
| 3rd test | 16 | 17 | 18 |
| 4th test | 19 | 20 | 21 |
| 5th test | 22 | 23 | 24 |
| 6th test | 25 | 26 | 27 |
| 7th test | 28 | 29 | 30 |
| 8th test | 31 | 32 | 33 |
| 9th test | 34 | 35 | 36 |
| 10th test | 37 | 38 | 39 |
| 11th test | 40 | 41 | 42 |
| 12th test | 43 | 44 | 45 |
| 13th test | 46 | 47 | 48 |
| 14th test | 49 | 50 | 51 |
| 15th test | 52 | 53 | 54 |
| 16th test | 55 | 56 | 57 |
| 17th test | 58 | 59 | 60 |
| 18th test | 61 | 62 | 63 |
| 19th test | 64 | 65 | 66 |
| 20th test | 67 | 68 | 69 |
| 21st test | 70 | 71 | 72 |
| 22nd test | 73 | 74 | 75 |
| 23rd test | 76 | 77 | 78 |
| 24th test | 79 | 80 | 81 |
| 25th test | 82 | 83 | 84 |
| 26th test | 85 | 86 | 87 |
| 27th test | 88 | 89 | 90 |
| 28th test | 91 | 92 | 93 |
| 29th test | 94 | 95 | 96 |
| 30th test | 97 | 98 | 99 |
| 31st test | 100 | 101 | 102 |
| 32nd test | 103 | 104 | 105 |
| 33rd test | 106 | 107 | 108 |
| 34th test | 109 | 110 | 111 |
| 35th test | 112 | 113 | 114 |
| 36th test | 115 | 116 | 117 |
| 37th test | 118 | 119 | 120 |
| 38th test | 121 | 122 | 123 |
| 39th test | 124 | 125 | 126 |
| 40th test | 127 | 128 | 129 |
| 41st test | 130 | 131 | 132 |
| 42nd test | 133 | 134 | 135 |
| 43rd test | 136 | 137 | 138 |
| 44th test | 139 | 140 | 141 |
| 45th test | 142 | 143 | 144 |
| 46th test | 145 | 146 | 147 |
| 47th test | 148 | 149 | 150 |
| 48th test | 151 | 152 | 153 |
| 49th test | 154 | 155 | 156 |
| 50th test | 157 | 158 | 159 |
| 51st test | 160 | 161 | 162 |
| 52nd test | 163 | 164 | 165 |
| 53rd test | 166 | 167 | 168 |
| 54th test | 169 | 170 | 171 |
| 55th test | 172 | 173 | 174 |
| 56th test | 175 | 176 | 177 |
| 57th test | 178 | 179 | 180 |
| 58th test | 181 | 182 | 183 |
| 59th test | 184 | 185 | 186 |
| 60th test | 187 | 188 | 189 |
| 61st test | 190 | 191 | 192 |
| 62nd test | 193 | 194 | 195 |
| 63rd test | 196 | 197 | 198 |
| 64th test | 199 | 200 | 201 |
| 65th test | 202 | 203 | 204 |
| 66th test | 205 | 206 | 207 |
| 67th test | 208 | 209 | 210 |
| 68th test | 211 | 212 | 213 |
| 69th test | 214 | 215 | 216 |
| 70th test | 217 | 218 | 219 |
| 71st test | 220 | 221 | 222 |
| 72nd test | 223 | 224 | 225 |
| 73rd test | 226 | 227 | 228 |
| 74th test | 229 | 230 | 231 |
| 75th test | 232 | 233 | 234 |
| 76th test | 235 | 236 | 237 |
| 77th test | 238 | 239 | 240 |
| 78th test | 241 | 242 | 243 |
| 79th test | 244 | 245 | 246 |
| 80th test | 247 | 248 | 249 |
| 81st test | 250 | 251 | 252 |
| 82nd test | 253 | 254 | 255 |
| 83rd test | 256 | 257 | 258 |
| 84th test | 259 | 260 | 261 |
| 85th test | 262 | 263 | 264 |
| 86th test | 265 | 266 | 267 |
| 87th test | 268 | 269 | 270 |
| 88th test | 271 | 272 | 273 |
| 89th test | 274 | 275 | 276 |
| 90th test | 277 | 278 | 279 |
| 91st test | 280 | 281 | 282 |
| 92nd test | 283 | 284 | 285 |
| 93rd test | 286 | 287 | 288 |
| 94th test | 289 | 290 | 291 |
| 95th test | 292 | 293 | 294 |
| 96th test | 295 | 296 | 297 |
| 97th test | 298 | 299 | 300 |
| 98th test | 301 | 302 | 303 |
| 99th test | 304 | 305 | 306 |
| 100th test | 307 | 308 | 309 |
| 101st test | 310 | 311 | 312 |
| 102nd test | 313 | 314 | 315 |
| 103rd test | 316 | 317 | 318 |
| 104th test | 319 | 320 | 321 |
| 105th test | 322 | 323 | 324 |
| 106th test | 325 | 326 | 327 |
| 107th test | 328 | 329 | 330 |
| 108th test | 331 | 332 | 333 |
| 109th test | 334 | 335 | 336 |
| 110th test | 337 | 338 | 339 |
| 111th test | 340 | 341 | 342 |
| 112th test | 343 | 344 | 345 |
| 113th test | 346 | 347 | 348 |
| 114th test | 349 | 350 | 351 |
| 115th test | 352 | 353 | 354 |
| 116th test | 355 | 356 | 357 |
| 117th test | 358 | 359 | 360 |
| 118th test | 361 | 362 | 363 |
| 119th test | 364 | 365 | 366 |
| 120th test | 367 | 368 | 369 |
| 121st test | 370 | 371 | 372 |
| 122nd test | 373 | 374 | 375 |
| 123rd test | 376 | 377 | 378 |
| 124th test | 379 | 380 | 381 |
| 125th test | 382 | 383 | 384 |
| 126th test | 385 | 386 | 387 |
| 127th test | 388 | 389 | 390 |
| 128th test | 391 | 392 | 393 |
| 129th test | 394 | 395 | 396 |
| 130th test | 397 | 398 | 399 |
| 131st test | 400 | 401 | 402 |
| 132nd test | 403 | 404 | 405 |
| 133rd test | 406 | 407 | 408 |
| 134th test | 409 | 410 | 411 |
| 135th test | 412 | 413 | 414 |
| 136th test | 415 | 416 | 417 |
| 137th test | 418 | 419 | 420 |
| 138th test | 421 | 422 | 423 |
| 139th test | 424 | 425 | 426 |
| 140th test | 427 | 428 | 429 |
| 141st test | 430 | 431 | 432 |
| 142nd test | 433 | 434 | 435 |
| 143rd test | 436 | 437 | 438 |
| 144th test | 439 | 440 | 441 |
| 145th test | 442 | 443 | 444 |
| 146th test | 445 | 446 | 447 |
| 147th test | 448 | 449 | 450 |
| 148th test | 451 | 452 | 453 |
| 149th test | 454 | 455 | 456 |
| 150th test | 457 | 458 | 459 |
| 151st test | 460 | 461 | 462 |
| 152nd test | 463 | 464 | 465 |
| 153rd test | 466 | 467 | 468 |
| 154th test | 469 | 470 | 471 |
| 155th test | 472 | 473 | 474 |
| 156th test | 475 | 476 | 477 |
| 157th test | 478 | 479 | 480 |
| 158th test | 481 | 482 | 483 |
| 159th test | 484 | 485 | 486 |
| 160th test | 487 | 488 | 489 |
| 161st test | 490 | 491 | 492 |
| 162nd test | 493 | 494 | 495 |
| 163rd test | 496 | 497 | 498 |
| 164th test | 499 | 500 | 501 |
| 165th test | 502 | 503 | 504 |
| 166th test | 505 | 506 | 507 |
| 167th test | 508 | 509 | 510 |
| 168th test | 511 | 512 | 513 |
| 169th test | 514 | 515 | 516 |
| 170th test | 517 | 518 | 519 |
| 171st test | 520 | 521 | 522 |
| 172nd test | 523 | 524 | 525 |
| 173rd test | 526 | 527 | 528 |
| 174th test | 529 | 530 | 531 |
| 175th test | 532 | 533 | 534 |
| 176th test | 535 | 536 | 537 |
| 177th test | 538 | 539 | 540 |
| 178th test | 541 | 542 | 543 |
| 179th test | 544 | 545 | 546 |
| 180th test | 547 | 548 | 549 |
| 181st test | 550 | 551 | 552 |
| 182nd test | 553 | 554 | 555 |
| 183rd test | 556 | 557 | 558 |
| 184th test | 559 | 560 | 561 |
| 185th test | 562 | 563 | 564 |
| 186th test | 565 | 566 | 567 |
| 187th test | 568 | 569 | 570 |
| 188th test | 571 | 572 | 573 |
| 189th test | 574 | 575 | 576 |
| 190th test | 577 | 578 | 579 |
| 191st test | 580 | 581 | 582 |
| 192nd test | 583 | 584 | 585 |
| 193rd test | 586 | 587 | 588 |
| 194th test | 589 | 590 | 591 |
| 195th test | 592 | 593 | 594 |
| 196th test | 595 | 596 | 597 |
| 197th test | 598 | 599 | 600 |
| 198th test | 601 | 602 | 603 |
| 199th test | 604 | 605 | 606 |
| 200th test | 607 | 608 | 609 |
| 201st test | 610 | 611 | 612 |
| 202nd test | 613 | 614 | 615 |
| 203rd test | 616 | 617 | 618 |
| 204th test | 619 | 620 | 621 |
| 205th test | 622 | 623 | 624 |
| 206th test | 625 | 626 | 627 |
| 207th test | 628 | 629 | 630 |
| 208th test | 631 | 632 | 633 |
| 209th test | 634 | 635 | 636 |
| 210th test | 637 | 638 | 639 |
| 211st test | 640 | 641 | 642 |
| 212nd test | 643 | 644 | 645 |
| 213rd test | 646 | 647 | 648 |
| 214th test | 649 | 650 | 651 |
| 215th test | 652 | 653 | 654 |
| 216th test | 655 | 656 | 657 |
| 217th test | 658 | 659 | 660 |
| 218th test | 661 | 662 | 663 |
| 219th test | 664 | 665 | 666 |
| 220th test | 667 | 668 | 669 |
| 221st test | 670 | 671 | 672 |
| 222nd test | 673 | 674 | 675 |
| 223rd test | 676 | 677 | 678 |
| 224th test | 679 | 680 | 681 |
| 225th test | 682 | 683 | 684 |
| 226th test | 685 | 686 | 687 |
| 227th test | 688 | 689 | 690 |
| 228th test | 691 | 692 | 693 |
| 229th test | 694 | 695 | 696 |
| 230th test | 697 | 698 | 699 |
| 231st test | 700 | 701 | 702 |
| 232nd test | 703 | 704 | 705 |
| 233rd test | 706 | 707 | 708 |
| 234th test | 709 | 710 | 711 |
| 235th test | 712 | 713 | 714 |
| 236th test | 715 | 716 | 717 |
| 237th test | 718 | 719 | 720 |
| 238th test | 721 | 722 | 723 |
| 239th test | 724 | 725 | 726 |
| 240th test | 727 | 728 | 729 |
| 241st test | 730 | 731 | 732 |
| 242nd test | 733 | 734 | 735 |
| 243rd test | 736 | 737 | 738 |
| 244th test | 739 | 740 | 741 |
| 245th test | 742 | 743 | 744 |
| 246th test | 745 | 746 | 747 |
| 247th test | 748 | 749 | 750 |
| 248th test | 751 | 752 | 753 |
| 249th test | 754 | 755 | 756 |
| 250th test | 757 | 758 | 759 |
| 251st test | 760 | 761 | 762 |
| 252nd test | 763 | 764 | 765 |
| 253rd test | 766 | 767 | 768 |
| 254th test | 769 | 770 | 771 |
| 255th test | 772 | 773 | 774 |
| 256th test | 775 | 776 | 777 |
| 257th test | 778 | 779 | 780 |
| 258th test | 781 | 782 | 783 |
| 259th test | 784 | 785 | 786 |
| 260th test | 787 | 788 | 789 |
| 261st test | 790 | 791 | 792 |
| 262nd test | 793 | 794 | 795 |
| 263rd test | 796 | 797 | 798 |
| 264th test | 799 | 800 | 801 |
| 265th test | 802 | 803 | 804 |
| 266th test | 805 | 806 | 807 |
| 267th test | 808 | 809 | 810 |
| 268th test | 811 | 812 | 813 |
| 269th test | 814 | 815 | 816 |
| 270th test | 817 | 818 | 819 |
| 271st test | 820 | 821 | 822 |
| 272nd test | 823 | 824 | 825 |
| 273rd test | 826 | 827 | 828 |
| 274th test | 829 | 830 | 831 |
| 275th test | 832 | 833 | 834 |
| 276th test | 835 | 836 | 837 |
| 277th test | 838 | 839 | 840 |
| 278th test | 841 | 842 | 843 |
| 279th test | 844 | 845 | 846 |
| 280th test | 847 | 848 | 849 |
| 281st test | 850 | 851 | 852 |
| 282nd test | 853 | 854 | 855 |
| 283rd test | 856 | 857 | 858 |
| 284th test | 859 | 860 | 861 |
| 285th test | 862 | 863 | 864 |
| 286th test | 865 | 866 | 867 |
| 287th test | 868 | 869 | 870 |
| 288th test | 871 | 872 | 873 |
| 289th test | 874 | 875 | 876 |
| 290th test | 877 | 878 | 879 |
| 291st test | 880 | 881 | 882 |
| 292nd test | 883 | 884 | 885 |
| 293rd test | 886 | 887 | 888 |
| 294th test | 889 | 890 | 891 |
| 295th test | 892 | 893 | 894 |
| 296th test | 895 | 896 | 897 |
| 297th test | 898 | 899 | 900 |
| 298th test | 901 | 902 | 903 |
| 299th test | 904 | 905 | 906 |
| 300th test | 907 | 908 | 909 |
| 301st test | 910 | 911 | 912 |
| 302nd test | 913 | 914 | 915 |
| 303rd test | 916 | 917 | 918 |
| 304th test | 919 | 920 | 921 |
| 305th test | 922 | 923 | 924 |
| 306th test | 925 | 926 | 927 |

9. Step Type Indication: Spread = 1. us.; Continuous 3 trials

| Test | A | B |
|----------|-------------|-------------|
| Settings | 71.0 | 66.0 |
| | 73.0 | 67.0 |
| | 68.0 | 67.0 |
| | 70.0 | 69.0 |
| | <u>67.0</u> | <u>69.0</u> |

Probable Error $1.3^\circ = 0.013 \text{ us.}$

10. Step Type Indication: Spread = 1.2 us.; Continuous Signals

| Test | A | B |
|----------|-------------|-------------|
| Settings | 62.0 | 56.0 |
| | 64.0 | 57.0 |
| | 65.0 | 53.0 |
| | 66.0 | 53.0 |
| | <u>66.0</u> | <u>52.0</u> |

Probable Error $1.3^\circ = 0.013 \text{ us.}$

11. Step Type Indication: Spread = 0.5 us.; Continuous Signals

| Test | A | B |
|------|--------------|--------------|
| | 240.0 | 238.0 |
| | 245.0 | 240.0 |
| | 246.0 | 242.0 |
| | 245.0 | 238.0 |
| | <u>243.0</u> | <u>237.0</u> |

Sweep Length 8 us.

Probable Error $1.4^\circ = 0.014 \text{ us.}$

CONSOLIDATED DATA FOR TEST 1. POTENTIAL DIFFERENCES
WITH 1.5. LINE

1. Receiver 52 feet from 10 cm. and 64 feet from 3 cm.

| Test | A | B | |
|----------|----------------|----------------|---|
| Shaft | 105.0
100.0 | 105.0
107.0 | Match obtained by
manipulation of
potentiometer |
| Settings | 97.0
105.0 | 105.0
101.0 | |
| Degrees | <u>103.0</u> | <u>103.0</u> | |

Probable Error $2.03^{\circ} = 0.0203 \text{ us.} = 10.0 \text{ feet}$

2. Time delay set on timer, receiver repeatedly moved for match

| Test | A | B |
|-----------|--------------|--------------|
| Distance, | 23.0
13.0 | 40.0
32.0 |
| Feet from | 18.5
14.5 | 27.0
53.0 |
| 10 cm. | <u>16.5</u> | <u>36.5</u> |

Probable Error $6.71 \text{ feet} = 0.014 \text{ us.}$

3. Receiver Stationary, Potentiometer Shaft Manipulated for match

| Test | A | B | C | D | E | F |
|-----------------------|------------------------|----------------|----------------|----------------|----------------|----------------|
| Shaft | 138
131 | 200
209 | 196
194 | 296
294 | 340
336 | 218
272 |
| Setting | 127
129 | 202
200 | 204
196 | 293
295 | 335
335 | 276
277 |
| Degrees | <u>132</u> | <u>(184)</u> | <u>196</u> | <u>295</u> | <u>(352)</u> | <u>279</u> |
| Probable Error
us. | 2.81°
0.03 | 2.94
0.03 | 2.78
0.03 | 0.89
0.01 | 1.92
0.02 | 2.65
0.03 |

| Test | 1 | 2 | 3 | 4 | 5 |
|--------------|-------|-------|-------|-------|-------|
| Distance | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Speed | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Acceleration | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Deceleration | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |

Probable error: 1.0% (1.0% of 100.0)

2. Time delay set on timer, recorded separately from test.

| Test | 1 | 2 | 3 | 4 | 5 |
|--------------|-------|-------|-------|-------|-------|
| Distance | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Speed | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Acceleration | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Deceleration | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |

Probable error: 1.0% (1.0% of 100.0)

3. Acceleration, constant, recorded separately from test.

| Test | 1 | 2 | 3 | 4 | 5 |
|----------------|-------|-------|-------|-------|-------|
| Distance | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Speed | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Acceleration | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Deceleration | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |
| Probable error | 1.0% | 1.0% | 1.0% | 1.0% | 1.0% |

A.6 Supplementary Discussion

A.6.1 Types of Signals Us able for System: In the body of the report, a pulse modulated system was described, other types of amplitude modulation could have been used. Let us consider a sinusoidally modulated system. In such a system, the modulation frequency must be low--so that the transverse error allowable in the system. This restriction is required to eliminate ambiguity in definition of the track line. The wavelength, therefore, would have to be in the order of 500 to 1,000 meters, or 600 to 300 K.C. The phase difference between signals from the Fixed and Rotating stations would indicate lines of position. This type of system would have definite advantages over the pulse system at the receiving end at least. Bandwidth of the receiver would be reduced by a factor of 1,000, and comparably better signal to noise ratios might be expected. This would not mean total gain to the system. Available transmitter power would be reduced in almost the same ratios as bandwidth of receiver is increased. Further, sinusoidal modulation is more difficult to apply to microwave transmitters than is pulse modulation. For these reasons, a pulse system was chosen for study.

A frequency modulated system could be used. In such a system the frequency of the transmitters is varied linearly (or sinusoidally) with time, repeating the variation periodically. The frequency difference received would then determine the line of position. This type of system would use relatively new techniques of modulating microwave transmitters. The timer of such a system could be simply a phase shifting transformer (when sinusoidal modulation is used). The receiver could use intermediate frequency amplification tuned to the difference between the carrier frequencies of the fixed and rotating stations. The indicator could be a

simple cycle rate counter with a dial indication. On the surface this appears to be the best system for development. There is one difficulty in the use of such a system. The carrier frequency difference between the two stations must be controlled to tolerances less than the frequency differences to be read. This would mean control within 20 cycles out of 10,000 MC., or 2 parts in 10^9 . While such control might be possible, it is very difficult to obtain. For this reason this type of system was not studied.

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A microwave, direction-
modulated, hyperbolic
navigational system.

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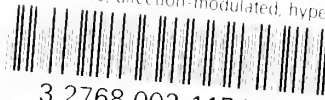
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